

ТИПОВОЙ РАСЧЁТ №2
«ИНТЕГРАЛЬНОЕ ИСЧИСЛЕНИЕ ФУНКЦИИ ОДНОЙ ПЕРЕМЕННОЙ.
ФУНКЦИИ НЕСКОЛЬКИХ ПЕРЕМЕННЫХ. ДИФФЕРЕНЦИАЛЬНЫЕ
УРАВНЕНИЯ И СИСТЕМЫ. ОПЕРАЦИОННОЕ ИСЧИСЛЕНИЕ»

ЗАДАНИЯ

РАЗДЕЛ 1. ДИФФЕРЕНЦИАЛЬНОЕ ИСЧИСЛЕНИЕ ФУНКЦИИ НЕСКОЛЬКИХ ПЕРЕМЕННЫХ.

1. Задана функция двух переменных $z = f(x; y)$. Найти: а) полные дифференциалы функции первого и второго порядка в произвольной точке; б) приближенное значение функции в точке $M(x; y)$, исходя из ее значения в точке $M_0(x_0; y_0)$, заменив приращение функции при переходе от точки M к точке M_0 полным дифференциалом; в) уравнения касательной плоскости и нормали к поверхности $z = f(x; y)$ в точке $M'_0(x_0; y_0; z_0)$, где $z_0 = f(x_0; y_0)$. Вычисления производить до третьего знака после запятой.

1.1 $z = y^3 \cdot e^{x^2}$, $M(0,02; 1,98)$, $M_0(0; 2)$.

1.2 $z = \frac{y^2 + x^2}{xy}$, $M(3,97; 3,03)$, $M_0(4; 3)$.

1.3 $z = y^2 \cdot \ln x$, $M(0,99; 2,01)$, $M_0(1; 2)$.

1.4 $z = 2x^4 \cdot \ln y$, $M(2,99; 0,98)$, $M_0(3; 1)$.

1.5 $z = y^2 \cdot x + 5 \cdot x^3 \cdot y^3$, $M(2,95; 3,05)$, $M_0(3; 3)$.

1.6 $z = x \cdot \arctg y$, $M(2,97; 0,01)$, $M_0(3; 0)$.

1.7 $z = y \cdot \arccos x$, $M(0,51; 4,99)$, $M_0(0,5; 5)$.

1.8 $z = \arcsin(y \cdot x)$, $M(0,52; 0,49)$, $M_0(0,5; 0,5)$.

1.9 $z = \frac{x}{y}$, $M(3,98; 2,01)$, $M_0(4; 2)$.

1.10 $z = \ln(x \cdot y)$, $M(0,96; 1,04)$, $M_0(1; 1)$.

1.11 $z = y^3 \cdot x^4 - x^3 \cdot y^4$, $M(2,93; 4,02)$, $M_0(3; 4)$.

1.12 $z = \arctg(x \cdot y)$, $M(1,03; 0,04)$, $M_0(1; 0)$.

1.13 $z = y^5 \cdot e^{-x}$, $M(0,07; 2,02)$, $M_0(0; 2)$.

1.14 $z = \ln(2x + 3y)$, $M(-0,99; 1,01)$, $M_0(-1; 1)$.

1.15 $z = e^{x \cdot y}$, $M(0,05; 1,97)$, $M_0(0; 2)$.

1.16 $z = \frac{1}{x^2 - y^2}$, $M(5,02; 2,98)$, $M_0(5; 3)$.

1.17 $z = e^{5x+4y}$, $M(1,05; -0,96)$, $M_0(1; -1)$.

1.18 $z = x \cdot \arcsin y$, $M(2,04; 0,02)$, $M_0(2; 0)$.

1.19 $z = y^x$, $M(1,03; 2,97)$, $M_0(1; 3)$.

1.20 $z = y \cdot \arcsin x$, $M(0,54; 0,46)$, $M_0(0,5; 0,5)$.

1.21 $z = \ln(x - y)$, $M(2,03; 0,96)$, $M_0(2; 1)$.

1.22 $z = x^4 \cdot e^{-2y}$, $M(1,96; 0,02)$, $M_0(2; 0)$.

1.23 $z = 2x^3 \cdot y^3 - 4 \cdot x^4 \cdot y^4$, $M(2,03; 1,97)$, $M_0(2; 2)$.

- 1.24 $z = \frac{1}{x^2 + y^2}$, $M(2,01;2,96)$, $M_0(2;3)$.
- 1.25 $z = y \cdot \arctg x$, $M(0,97;2,04)$, $M_0(1;2)$.
- 1.26 $z = \frac{1}{\sqrt{x^2 + y^2}}$, $M(3,01;3,97)$, $M_0(3;4)$.
- 1.27 $z = x \cdot y^2 - x^2 \cdot y$, $M(1,97;0,98)$, $M_0(2;1)$.
- 1.28 $z = \frac{y}{x}$, $M(2,97;6,01)$, $M_0(3;6)$.
- 1.29 $z = \frac{1}{\sqrt{x^2 - y^2}}$, $M(13,01;11,99)$, $M_0(13;12)$.
- 1.30 $z = x^2 \cdot y^6 - 2 \cdot x^6 \cdot y^2$, $M(2,01;1,98)$, $M_0(2;2)$.

2. Исследовать на экстремум функцию двух переменных $z = f(x; y)$.

- 2.1. $z = x^3 + y^2x - 3xy$. 2.2. $z = y^3 + x^2y - 3yx$. 2.3. $z = 12y^3 + 18x^3 - 2yx$.
- 2.4. $z = 18y^3 + 12x^3 - 2yx$. 2.5. $z = 8y^3 + x^3 - 6yx - 9$.
- 2.6. $z = 2y^3 + 2x^3 - 6yx - 4$. 2.7. $z = 3y^3 + 3x^3 - 9yx + 9$.
- 2.8. $z = y^3 + 3x^2y - 15y - 12x$. 2.9. $z = y^2 + x^2 + yx + x - y + 2$.
- 2.10. $z = -2y^2 - 3x^2 + 2yx - 5$. 2.11. $z = 8x - 8y - y^2 - x^2 - 7$
- 2.12. $z = 2xy - \frac{2}{3}x^3 - y^2 - 12$. 2.13. $z = -y^4 - x^4 - 2x^2 - 2y^2 + 4yx + 2$.
- 2.14. $z = y^3 + x^3 - 6yx + 1$. 2.15. $z = y^2 + x^2 - 2x + 4y + 3$.
- 2.16. $z = x^2y + xy^2 - yx + 4$. 2.17. $z = y^2 + x^2 + yx - 13x - 11y + 5$.
- 2.18. $z = y^3 + x^3 + 6yx - 2$. 2.19. $z = x^3 - y^2 - 7x^2 + yx + 9x + 3y - 9$.
- 2.20. $z = 2x^3 + 2y^3 - 14xy + 5$. 2.21. $z = y^3 - x^2 - 7y^2 + xy + 9y + 3x + 5$.
- 2.22. $z = 7 - 6y^2 - 10x^2 + 4yx$. 2.23. $z = 5 - 2x\sqrt{y} + 2x^2 + 2y - 12x$.
- 2.24. $z = 3 + 2x^2y + 2y^2x - 24xy$. 2.25. $z = 3y^2 + 3x^2 - 6x + 12y - 1$.
- 2.26. $z = 3y^3 + 3x^3 + 18yx - 7$. 2.27. $z = 4 - 3y\sqrt{x} + 3y^2 + 3x - 18y$.
- 2.28. $z = 2xy - x^3 - y^2x$. 2.29. $z = 9xy - 3y^3 - 3x^2y$. 2.30. $z = y^3 + x^3 + 9yx$.

3. Найти условный экстремум функции $z = f(x; y)$ при условии $\varphi(x; y) = 0$.

- 3.1. $f(x; y) = \frac{1}{x} + \frac{1}{y}$, $\varphi(x; y) = x + y - 2$. 3.2. $f(x; y) = xy^2$, $\varphi(x; y) = x + 2y - 1$.
- 3.3. $f(x; y) = x + y$, $\varphi(x; y) = x^2 + y^2 - 1$. 3.4. $f(x; y) = \frac{x-y}{\sqrt{2}}$, $\varphi(x; y) = x^2 + y^2 - 1$.
- 3.5. $f(x; y) = x^2 + y^2 - xy + x + y - 4$, $\varphi(x; y) = x + y + 3$.

- 3.6. $f(x; y) = x^2 + y^2 + xy - 5x - 4y + 10$, $\varphi(x; y) = x + y - 4$.
- 3.7. $f(x; y) = x^2 - y^2$, $\varphi(x; y) = x + 2y - 6$. 3.8. $f(x; y) = x^2 y$, $\varphi(x; y) = 2x + y - 1$.
- 3.9. $f(x; y) = x^2 + y^2$, $\varphi(x; y) = x + y - 2$ 3.10. $f(x; y) = xy$, $\varphi(x; y) = x^2 + y^2 - 2$.
- 3.11. $f(x; y) = \frac{1}{x} + \frac{1}{y}$, $\varphi(x; y) = \frac{1}{x^2} + \frac{1}{y^2} - \frac{1}{9}$.
- 3.12. $f(x; y) = \frac{1}{x} + \frac{1}{y}$, $\varphi(x; y) = x + y - 2$. 3.13. $f(x; y) = xy^2$, $\varphi(x; y) = x + 2y - 5$.
- 3.14. $f(x; y) = \cos^2 x + \cos^2 y$, $\varphi(x; y) = x - y + \frac{\pi}{4}$.
- 3.15. $f(x; y) = 2x + 2y$, $\varphi(x; y) = x^2 + y^2 - 8$.
- 3.16. $f(x; y) = 2x - 2y$, $\varphi(x; y) = x^2 + y^2 - 18$.
- 3.17. $f(x; y) = \frac{3}{x} + \frac{3}{y}$, $\varphi(x; y) = \frac{3}{x^2} + \frac{3}{y^2} - 6$.
- 3.18. $f(x; y) = \frac{1}{x} - \frac{1}{y}$, $\varphi(x; y) = \frac{1}{x^2} + \frac{1}{y^2} - \frac{1}{2}$.
- 3.19. $f(x; y) = x^2 - 2\sqrt{y}$, $\varphi(x; y) = x - y + 0,5$.
- 3.20. $f(x; y) = y^2 - 2\sqrt{x}$, $\varphi(x; y) = y - x - 0,75$.
- 3.21. $f(x; y) = \sin^2 x + \sin^2 y$, $\varphi(x; y) = x + y + \frac{3\pi}{4}$.
- 3.22. $f(x; y) = 3x^2 + 4y^2$, $\varphi(x; y) = 12x + 16y + 56$.
- 3.23. $f(x; y) = 2x + 4y$, $\varphi(x; y) = x^2 + y^2 - 20$.
- 3.24. $f(x; y) = \sin^2 x + \cos^2 y$, $\varphi(x; y) = x - y - \frac{5\pi}{4}$.
- 3.25. $f(x; y) = 2x^2 - 3y^2$, $\varphi(x; y) = 12(x - y) + 120$.
- 3.26. $f(x; y) = 3x + 5y$, $\varphi(x; y) = 9x^2 - 10y^2 + 54$.
- 3.27. $f(x; y) = \sin^2 x - \sin^2 y$, $\varphi(x; y) = x - y + \frac{7\pi}{4}$.
- 3.28. $f(x; y) = x^2 + 6y^2$, $\varphi(x; y) = 6x + 24y - 60$.
- 3.29. $f(x; y) = 2x + y$, $\varphi(x; y) = 4x^2 + y^2 - 8$.
- 3.30. $f(x; y) = \cos^2 x - \cos^2 y$, $\varphi(x; y) = x + y - \frac{\pi}{4}$.

4. Найти наибольшее и наименьшее значения функции $z = f(x; y)$ в замкнутой области G , заданной системой неравенств. Сделать чертёж.

- 4.1. $f(x; y) = x^2 + xy + x$, $x \leq 0, y \leq 0, x + y \geq -2$.
- 4.2. $f(x; y) = 2x^2 + 3xy - 7x - 3y$, $x \geq 0, y \geq 0, x + y \leq 3$.
- 4.3. $f(x; y) = 3x^2 + 4xy + 4y - 2x$, $x \leq 0, y \geq 0, y - x \leq 3$.
- 4.4. $f(x; y) = 4x^2 + 2xy - 16x - 6y$, $x \geq 0, y \leq 0, x - y \leq 8$.

- 4.5. $f(x; y) = x^2 + y^2 - 9xy + 27, \quad 0 \leq x \leq 3, 0 \leq y \leq 3.$
- 4.6. $f(x; y) = x^2 + 2y^2 + 1, \quad x \geq 0, y \geq 0, x + y \leq 3.$
- 4.7. $f(x; y) = 3 - 2x^2 - xy - y^2, \quad x \leq 1, y \geq 0, x - y \geq 0.$
- 4.8. $f(x; y) = x^2 + 3y^2 + x - y, \quad x \geq 1, y \geq -1, x + y \leq 1.$
- 4.9. $f(x; y) = 5x^2 + y^2 - 3xy + 4, \quad x \geq -1, y \geq -1, x + y \leq 1.$
- 4.10. $f(x; y) = x^2 - y^2 + 2xy + 4x, \quad x \leq 0, y \leq 0, x + y \geq -2.$
- 4.11. $f(x; y) = x^2 + xy - 2, \quad 4x^2 - 4 \leq y \leq 0.$
- 4.12. $f(x; y) = 10 + 4xy - x^2, \quad 0 \leq y \leq 4 - x^2.$
- 4.13. $f(x; y) = x^2 - y^2, \quad x^2 + y^2 \leq 9.$
- 4.14. $f(x; y) = x^2 + 2xy - 4x + 8y, \quad 0 \leq x \leq 1, 0 \leq y \leq 2.$
- 4.15. $f(x; y) = 4x^2y - x^3y - x^2y^2, \quad x \geq 0, y \geq 0, x + y \leq 6.$
- 4.16. $f(x; y) = (2x^2 + 3y^2) \cdot e^{-(x^2 + y^2)}, \quad x^2 + y^2 \leq 9.$
- 4.17. $f(x; y) = \sin x + \sin y + \sin(x + y), \quad 0 \leq x \leq \frac{\pi}{2}, 0 \leq y \leq \frac{\pi}{2}.$
- 4.18. $f(x; y) = 2x^2 - 2y^2, \quad x^2 + y^2 \leq 16.$
- 4.19. $f(x; y) = x^3 + y^3 + 6xy, \quad -3 \leq x \leq 1, -3 \leq y \leq 2.$
- 4.20. $f(x; y) = 2x^2y - x^3y - x^2y^2, \quad x \geq 0, y \geq 0, x + y \leq 6.$
- 4.21. $f(x; y) = 5x^2 - 3xy + y^2, \quad 0 \leq x \leq 1, 0 \leq y \leq 1.$
- 4.22. $f(x; y) = x^2 - y^2 + 2xy - 4x, \quad x \leq 3, y \geq 0, x - y \geq -1.$
- 4.23. $f(x; y) = 4 - 2x^2 - y^2, \quad y \geq 0, y \leq \sqrt{1 - x^2}.$
- 4.24. $f(x; y) = \cos x + \cos y + \cos(x + y), \quad 0 \leq x \leq \frac{\pi}{2}, 0 \leq y \leq \frac{\pi}{2}.$
- 4.25. $f(x; y) = 1 + 2x^2 + 3y^2, \quad y \geq 0, y \leq \sqrt{9 - \frac{9}{4}x^2}.$
- 4.26. $f(x; y) = \frac{1}{2}x^2 - xy, \quad 2x^2 \leq y \leq 8.$
- 4.27. $f(x; y) = xy, \quad x^2 + y^2 \leq 1.$
- 4.28. $f(x; y) = xy - 3x - 2y, \quad 0 \leq x \leq 4, 0 \leq y \leq 4.$
- 4.29. $f(x; y) = x^3 + y^3 - 3xy, \quad 0 \leq x \leq 2, -1 \leq y \leq 2.$
- 4.30. $f(x; y) = xy^2, \quad x^2 + y^2 \leq 1.$

РАЗДЕЛ 2. ИНТЕГРАЛЬНОЕ ИСЧИСЛЕНИЕ ФУНКЦИИ ОДНОЙ ПЕРЕМЕННОЙ

5. Найти неопределенные интегралы.

5.1. а) $\int \frac{dx}{x\sqrt{x^2+1}}$; б) $\int (4-3x)e^{-3x} dx$; в) $\int \frac{2x+1}{x^3+8} dx$; г) $\int \frac{\sqrt{9-x^2}}{x^2} dx$;

д) $\int \frac{dx}{3-4\cos x+\sin x}$; е) $\int \frac{dx}{(x+1)\sqrt{1-x-x^2}}$; ж) $\int \frac{\sin^2 x dx}{2\sin^2 x - \cos^2 x}$.

5.2. а) $\int \frac{1+\ln x}{x} dx$; б) $\int x^2 \sin x dx$; в) $\int \frac{3x^3-2}{x^3-x} dx$; г) $\int x^2 \sqrt{1-x^2} dx$;

д) $\int \frac{dx}{5\sin x-2\cos x+1}$; е) $\int \frac{dx}{(x-2)\sqrt{1-x-x^2}}$; ж) $\int \frac{tg x dx}{\sin^2 x + 4\cos^2 x}$.

5.3. а) $\int \frac{dx}{x\sqrt{x^2-1}}$; б) $\int x \arcsin x dx$; в) $\int \frac{3x^3-x^2-12x-2}{x(x+1)(x-2)} dx$; г) $\int \frac{\sqrt{9-x^2}}{x^4} dx$;

д) $\int \frac{dx}{3-5\cos x+6\sin x}$; е) $\int \frac{dx}{x\sqrt{1-x-x^2}}$; ж) $\int \frac{dx}{8-4\cos^2 x}$.

5.4. а) $\int x^2 \sqrt[5]{x^3+2} dx$; б) $\int (7x-10)\sin 5x dx$; в) $\int \frac{2x^3-x^2-7x-12}{x(x-3)(x+1)} dx$;

г) $\int x^3 \sqrt{16+x^2} dx$; д) $\int \frac{dx}{3\cos x-4\sin x}$; е) $\int \frac{dx}{x\sqrt{x^2+x-3}}$; ж) $\int \frac{\cos^2 x dx}{4-\sin^2 x}$.

5.5. а) $\int tg x \ln \cos x dx$; б) $\int (4-2x)\cos 5x dx$; в) $\int \frac{x^3-5x^2+5x+23}{(x-1)(x+1)(x-5)} dx$;

г) $\int \frac{x^2}{\sqrt{4+x^2}} dx$; д) $\int \frac{dx}{3+\cos x+\sin x}$; е) $\int \frac{dx}{(x-1)\sqrt{x^2+x-2}}$; ж) $\int \frac{dx}{2+5\sin^2 x}$.

5.6. а) $\int \frac{\sin x - \cos x}{(\cos x + \sin x)^4} dx$; б) $\int \arctg \sqrt{2x-1} dx$; в) $\int \frac{x^3-3x^2-12}{(x-4)(x-3)x} dx$;

г) $\int \frac{x-5}{\sqrt{x^2+25}} dx$; д) $\int \frac{dx}{4\sin x-2\cos x}$; е) $\int \frac{dx}{(x+1)\sqrt{x^2-3x+2}}$; ж) $\int \frac{dx}{4-3\sin^2 x}$.

5.7. а) $\int \frac{3+\ln(2x-1)}{2x-1} dx$; б) $\int (2x-3)\cos 4x dx$; в) $\int \frac{3x^3+2x^2+1}{(x+2)(x-2)^2} dx$;

г) $\int \frac{\sqrt{x^2-9}}{x^3} dx$; д) $\int \frac{dx}{3-4\cos x}$; е) $\int \frac{dx}{(x-3)\sqrt{2-x-x^2}}$; ж) $\int \frac{(2tg x - 1)dx}{\sin^2 x + 2\cos^2 x}$.

5.8. а) $\int \frac{1/(2\sqrt{x})+1}{(\sqrt{x}+x)^2} dx$; б) $\int (5x-2)e^{6x} dx$; в) $\int \frac{x^3-3x^2-12}{(x-4)(x-3)(x-2)} dx$;
г) $\int \frac{\sqrt{9-x^2}}{x} dx$; д) $\int \frac{3\sin x + \cos x}{1+\sin x} dx$; е) $\int \frac{dx}{(x+1)\sqrt{1-3x-2x^2}}$; ж) $\int \frac{dx}{2\cos^2 x + 5}$.

5.9. а) $\int \frac{x^3 dx}{\sqrt[3]{x^4+1}}$; б) $\int \ln(x^2+4) dx$; в) $\int \frac{4x^3+x^2+2}{x(x-1)(x-2)} dx$; г) $\int \frac{x^2}{\sqrt{4-x^2}} dx$;
д) $\int \frac{6+2\cos x}{2-\cos x+\sin x} dx$; е) $\int \frac{dx}{x\sqrt{1+x^2}}$; ж) $\int \frac{dx}{2\sin^2 x - \sin 2x + \cos^2 x}$.

5.10. а) $\int \frac{(6x-5)dx}{2\sqrt{3x^2-5x+6}}$; б) $\int \operatorname{arctg} \sqrt{5x-1} dx$; в) $\int \frac{x^3-3x^2-12}{x^2-4x} dx$; г) $\int \frac{\sqrt{9-x^2}}{x^2} dx$;
д) $\int \frac{dx}{2-\cos x+\sin x}$; е) $\int \frac{dx}{x\sqrt{x^2-1}}$; е) $\int \frac{dx}{x\sqrt{1+x+x^2}}$; ж) $\int \frac{dx}{3\sin^2 x - 16\cos^2 x}$.

5.11. а) $\int \frac{\sqrt{\ln x}}{x} dx$; б) $\int \frac{x}{\sin^2 x} dx$; в) $\int \frac{2x^3-40x-8}{x(x+4)(x-2)} dx$; г) $\int \frac{x^2}{\sqrt{9-x^2}} dx$;
д) $\int \frac{dx}{5+\cos x+2\sin x}$; е) $\int \frac{dx}{(x-1)\sqrt{x^2-x+1}}$; ж) $\int \frac{\sin 2x dx}{\sin^4 x + 2\cos^4 x}$.

5.12. а) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}} dx$; б) $\int xe^{-x} dx$; в) $\int \frac{x^3}{(x-1)(x+1)(x+2)} dx$; г) $\int \frac{dx}{(9+x^2)\sqrt{9+x^2}}$;
д) $\int \frac{2-\sin x+2\cos x}{1+\cos x} dx$; е) $\int \frac{dx}{x\sqrt{x+x^2-1}}$; ж) $\int \frac{dx}{5\sin^2 x - 7\cos^2 x}$.

5.13. а) $\int \frac{dx}{\cos^2 x \sqrt{1+\operatorname{tg} x}}$; б) $\int (1-4x)\cos 3x dx$; в) $\int \frac{x^3+2x^2+3}{(x-1)(x-2)(x-3)} dx$;
г) $\int \frac{dx}{(16+x^2)^{3/2}}$; д) $\int \frac{dx}{1-4\cos x+2\sin x}$; е) $\int \frac{dx}{(x-3)\sqrt{1+x-x^2}}$; ж) $\int \frac{dx}{3\cos^2 x - 4}$.

5.14. а) $\int e^x(\sin e^x) dx$; б) $\int \frac{x}{\cos^2 x} dx$; в) $\int \frac{x^2+1}{x^3-27} dx$; г) $\int x^2\sqrt{9-x^2} dx$;
д) $\int \frac{dx}{\sin x - 3\cos x}$; е) $\int \frac{dx}{(x-1)\sqrt{x^2+x-2}}$; ж) $\int \frac{\sin 2x dx}{4\sin^4 x + \cos^4 x}$.

5.15. а) $\int e^{-x^3} x^2 dx$; б) $\int (3-7x)e^{7x} dx$; в) $\int \frac{x}{64-x^3} dx$; г) $\int x^2\sqrt{25-x^2} dx$;
д) $\int \frac{dx}{6\cos x + \sin x}$; е) $\int \frac{dx}{(x+1)\sqrt{x^2-x+1}}$; ж) $\int \frac{dx}{2+\sin^2 x}$.

$$5.16. \text{ а) } \int \frac{1 - \cos x}{(x - \sin x)^2} dx; \text{ б) } \int (4 - 7x) \sin 6x dx; \text{ в) } \int \frac{x^3 + x + 2}{(x + 2)(x + 1)^2} dx; \text{ г) } \int \frac{dx}{\sqrt{(1 + x^2)^3}};$$

$$\text{ д) } \int \frac{\sin x}{5 + 3 \sin x} dx; \text{ е) } \int \frac{dx}{(x - 3)\sqrt{x^2 - x - 1}}; \text{ ж) } \int \frac{dx}{\sin^3 x \cos x}.$$

$$5.17. \text{ а) } \int \frac{1 + \ln(x - 1)}{x - 1} dx; \text{ б) } \int \operatorname{arctg} \sqrt{3x - 1} dx; \text{ в) } \int \frac{x^3 - 1}{(x + 1)^2(x - 2)} dx; \text{ г) } \int \frac{dx}{x\sqrt{x^2 - 1}};$$

$$\text{ д) } \int \frac{\cos x}{2 + \cos x} dx; \text{ е) } \int \frac{dx}{(x + 2)\sqrt{x^2 + x + 1}}; \text{ ж) } \int \frac{dx}{2 \sin^2 x + 7 \cos^2 x}.$$

$$5.18. \text{ а) } \int \frac{x + \cos x}{x^2 + 2 \sin x} dx; \text{ б) } \int (-3 - 6x) \cos 8x dx; \text{ в) } \int \frac{x^3 - 2x^2 + x}{(x - 1)^3} dx;$$

$$\text{ г) } \int \frac{x^4 dx}{(16 - x^2)\sqrt{16 - x^2}}; \text{ д) } \int \frac{dx}{2 - \sin x}; \text{ е) } \int \frac{dx}{(x - 3)\sqrt{x^2 + x - 1}}; \text{ ж) } \int \frac{dx}{4 \sin^2 x - 7 \cos^2 x}.$$

$$5.19. \text{ а) } \int \frac{(\arcsin x)^2 + 1}{\sqrt{1 - x^2}} dx; \text{ б) } \int (x - 7) \sin 9x dx; \text{ в) } \int \frac{1 + x^2}{(1 + x)^2(x - 1)} dx; \text{ г) } \int \frac{dx}{x\sqrt{x^2 - 1}};$$

$$\text{ д) } \int \frac{dx}{1 + \cos x + 3 \sin x}; \text{ е) } \int \frac{dx}{(x + 4)\sqrt{1 + x - x^2}}; \text{ ж) } \int \frac{\operatorname{tg} x dx}{1 - \operatorname{ctg}^2 x}.$$

$$5.20. \text{ а) } \int \frac{\operatorname{arctg} x + x}{1 + x^2} dx; \text{ б) } \int \arccos 2x dx; \text{ в) } \int \frac{3x^3 + 25}{(x^2 + 3x + 2)(x - 1)} dx; \text{ г) } \int \frac{dx}{x^3 \sqrt{x^2 - 1}};$$

$$\text{ д) } \int \frac{dx}{5 + 3 \cos x}; \text{ е) } \int \frac{dx}{(x - 2)\sqrt{x^2 + x + 1}}; \text{ ж) } \int \frac{dx}{2 \sin^2 x + 9 \cos^2 x}.$$

$$5.21. \text{ а) } \int \frac{8x - \operatorname{arctg} 2x}{1 + 4x^2} dx; \text{ б) } \int \frac{\arcsin x}{\sqrt{x + 1}} dx; \text{ в) } \int \frac{2x - x^2}{(x + 2)^3} dx; \text{ г) } \int x^2 \sqrt{25 - x^2} dx;$$

$$\text{ д) } \int \frac{dx}{2 - 2 \cos x + \sin x}; \text{ е) } \int \frac{dx}{(x - 3)\sqrt{x^2 - x + 1}}; \text{ ж) } \int \frac{(2 \operatorname{tg} x + 3) dx}{\sin^2 x + 2 \cos^2 x}.$$

$$5.22. \text{ а) } \int \cos^3 x \sin 2x dx; \text{ б) } \int x^2 \ln(1 + x) dx; \text{ в) } \int \frac{9 - x^2}{(3 + x)^3} dx; \text{ г) } \int \sqrt{25 - x^2} dx;$$

$$\text{ д) } \int \frac{\cos x dx}{1 + \cos x + \sin x}; \text{ е) } \int \frac{dx}{(x + 4)\sqrt{x^2 + x - 1}}; \text{ ж) } \int \frac{dx}{1 + 4 \cos^2 x}.$$

$$5.23. \text{ а) } \int \operatorname{ctg}(2x + 1) dx; \text{ б) } \int \arcsin 6x dx; \text{ в) } \int \frac{x^3 + 8}{x^3 - 27} dx; \text{ г) } \int \frac{x^4 dx}{\sqrt{(8 - x^2)^3}};$$

$$\text{ д) } \int \frac{\sin x dx}{1 + \cos x + \sin x}; \text{ е) } \int \frac{dx}{(x - 5)\sqrt{x^2 - x - 1}}; \text{ ж) } \int \frac{dx}{9 \sin^2 x - 8 \sin x \cos x}.$$

5.24. а) $\int \frac{e^{2x}}{e^{2x} + 9} dx$; б) $\int 5x \cos 3x dx$; в) $\int \frac{x+1}{(x^2+1)(x-2)} dx$; г) $\int \frac{dx}{\sqrt{(4-x^2)^3}}$;
 д) $\int \frac{dx}{8-4\sin x+7\cos x}$; е) $\int \frac{dx}{(x-6)\sqrt{1+x-x^2}}$; ж) $\int \frac{\sin^2 x dx}{\sin^2 x + 3\sin x \cos x - \cos^2 x}$.

5.25. а) $\int \frac{x - (\operatorname{arctg} x)^4}{1+x^2} dx$; б) $\int (4x-1)\sin(1-x) dx$; в) $\int \frac{1}{(x^2+x+1)(x+1)} dx$;
 г) $\int \frac{\sqrt{x^2-2}}{x^4} dx$; д) $\int \frac{dx}{3+3\cos x+2\sin x}$; е) $\int \frac{dx}{(x+4)\sqrt{x^2-3x+2}}$; ж) $\int \frac{dx}{6\sin^2 x+7}$.

5.26. а) $\int \frac{dx}{(\arcsin x)^3 \sqrt{1-x^2}}$; б) $\int (2x-4)e^{-8x} dx$; в) $\int \frac{1-2x}{(x^2+4)(x-1)} dx$; г) $\int \frac{\sqrt{x^2-1}}{x^4} dx$;
 д) $\int \frac{dx}{4\cos x+7\sin x}$; е) $\int \frac{dx}{(x-1)\sqrt{x^2+x-3}}$; ж) $\int \frac{dx}{6-5\cos^2 x}$.

5.27. а) $\int \frac{\sin^5 x}{\cos^4 x} dx$; б) $\int 3x \operatorname{arctg} 7x dx$; в) $\int \frac{x}{(x^2+9)(x-1)} dx$; г) $\int \sqrt{121-x^2} dx$;
 д) $\int \frac{2\sin x - \cos x}{4 - \cos x} dx$; е) $\int \frac{dx}{(x-6)\sqrt{1-3x-2x^2}}$; ж) $\int \frac{dx}{\sin^2 x + \sin 2x + 4\cos^2 x}$.

5.28. а) $\int \frac{\sin x}{\sqrt{1+6\cos x}} dx$; б) $\int \operatorname{arctg} \sqrt{8x-1} dx$; в) $\int \frac{3-x^2}{(x-1)(x+2)^2} dx$; г) $\int \frac{dx}{x^3 \sqrt{x^2-1}}$;
 д) $\int \frac{3\cos x - \sin x}{\cos x + \sin x} dx$; е) $\int \frac{dx}{(x+7)\sqrt{1-x-x^2}}$; ж) $\int \frac{\sin^2 x dx}{8\sin^2 x - 3\cos^2 x}$.

5.29. а) $\int \frac{x^2 + (\ln x)^2}{x} dx$; б) $\int \arcsin \sqrt{x} dx$; в) $\int \frac{x+6}{(x+1)(x-5)(x+2)} dx$; г) $\int \frac{x^2 dx}{\sqrt{36-x^2}}$;
 д) $\int \frac{\sin x dx}{8+\cos x}$; е) $\int \frac{dx}{(x-8)\sqrt{x^2+x-2}}$; ж) $\int \frac{\sin^2 x dx}{2\sin^2 x + 3\cos^2 x}$.

5.30. а) $\int \frac{4\operatorname{arctg} x - x}{1+x^2} dx$; б) $\int \ln(4x^2+1) dx$; в) $\int \frac{1-x}{(x^2+4x+5)(x-1)} dx$;
 г) $\int x^2 \sqrt{64-x^2} dx$; д) $\int \frac{\cos x dx}{7+\sin x}$; е) $\int \frac{dx}{(x-1)\sqrt{x^2-3x+2}}$; ж) $\int \frac{\cos^2 x dx}{2\sin^2 x + \cos^2 x}$.

6. Найти определенные интегралы.

6.1. а) $\int_0^1 \frac{2x dx}{1+\sqrt{x}}$; б) $\int_{-3}^0 (x^2+6x+9)\sin 2x dx$; в) $\int_0^{\pi/2} \frac{\sin x dx}{5+3\sin x}$; г) $\int_0^{\pi/3} \cos^3 x \sin 2x dx$.

6.2. а) $\int_2^9 \frac{x dx}{\sqrt[3]{x-1}}$; б) $\int_0^1 x^2 e^{3x} dx$; в) $\int_0^{\pi/2} \frac{\sin x dx}{2+\sin x}$; г) $\int_0^{\pi/2} \cos x \cos 3x \cos 5x dx$.

$$6.3. \text{ а) } \int_0^1 \frac{x^3 + x}{x^4 + 1} dx; \text{ б) } \int_{\pi/4}^{\pi/3} \frac{xdx}{\sin^2 x}; \text{ в) } \int_0^{\pi/4} \frac{dx}{\cos x(1 + \cos x)}; \text{ г) } \int_{\pi/4}^{\pi} \sin x \sin 2x \sin 3x dx.$$

$$6.4. \text{ а) } \int_{\sqrt{2}}^{\sqrt{3}} \frac{xdx}{\sqrt{x^4 - x^2 - 1}}; \text{ б) } \int_0^{e-1} \ln(x+1) dx; \text{ в) } \int_0^{\pi/2} \frac{dx}{(1 + \sin x + \cos x)^2}; \text{ г) } \int_0^{\pi/4} \sin 3x \cos 5x dx.$$

$$6.5. \text{ а) } \int_{\pi}^{2\pi} \frac{1 - \cos x}{(x - \sin x)^2} dx; \text{ б) } \int_1^e \sqrt{x} \ln^2 x dx; \text{ в) } \int_0^{2\pi/3} \frac{\cos^2 x dx}{(1 + \cos x + \sin x)^2}; \text{ г) } \int_0^{\pi} \cos \frac{x}{2} \cos \frac{3x}{2} dx.$$

$$6.6. \text{ а) } \int_{-1}^0 \frac{\operatorname{tg}(x+1)}{\cos^2(x+1)} dx; \text{ б) } \int_1^e \frac{\ln^2 x dx}{\sqrt{x}}; \text{ в) } \int_0^{2\pi/3} \frac{\sin^2 x dx}{(1 + \cos x + \sin x)^2}; \text{ г) } \int_0^{\pi/4} 2 \cos x \sin 3x dx.$$

$$6.7. \text{ а) } \int_0^1 \frac{xdx}{\sqrt{x^4 + x^2 + 1}}; \text{ б) } \int_{-1/3}^{-2/3} \frac{x}{e^{3x}} dx; \text{ в) } \int_{-2\pi/3}^0 \frac{\cos^2 x dx}{(1 + \cos x - \sin x)^2}; \text{ г) } \int_0^1 \sqrt{(1-x^2)^3} dx.$$

$$6.8. \text{ а) } \int_1^e \frac{x + \ln x^2}{x} dx; \text{ б) } \int_{-2}^0 (x^2 + 2)e^{x/2} dx; \text{ в) } \int_{-\pi/2}^0 \frac{\sin x dx}{(1 + \cos x - \sin x)^2}; \text{ г) } \int_0^{\pi} \cos \frac{x}{2} \cos \frac{x}{3} dx.$$

$$6.9. \text{ а) } \int_{\sqrt{2}}^2 \frac{dx}{x\sqrt{x^2 - 1}}; \text{ б) } \int_{-1}^1 x^2 e^{-x/2} dx; \text{ в) } \int_0^{\pi/2} \frac{\sin x dx}{(1 + \cos x + \sin x)^2}; \text{ г) } \int_0^{\pi} \cos^4 x \sin^2 x dx.$$

$$6.10. \text{ а) } \int_{\sqrt{3}}^{\sqrt{8}} \frac{dx}{x\sqrt{x^2 + 1}}; \text{ б) } \int_1^8 \frac{\ln^2 x dx}{\sqrt[3]{x^2}}; \text{ в) } \int_{-\pi/2}^0 \frac{\cos x dx}{(1 + \cos x - \sin x)^2}; \text{ г) } \int_{\pi/2}^{\pi} \cos^2 x \sin^4 x dx.$$

$$6.11. \text{ а) } \int_1^3 \frac{1 - \sqrt{x}}{\sqrt{x}(x+1)} dx; \text{ б) } \int_0^{\pi/2} (1 - 5x^2) \sin x dx; \text{ в) } \int_0^{\pi/2} \frac{\cos x dx}{(1 + \cos x + \sin x)^2}; \text{ г) } \int_0^{\pi} \sin^4 \frac{x}{2} dx.$$

$$6.12. \text{ а) } \int_0^{\sqrt{3}} \frac{x - (\operatorname{arctg} x)^4}{1 + x^2} dx; \text{ б) } \int_0^1 x e^{-x} dx; \text{ в) } \int_{-2\pi/3}^0 \frac{\cos x dx}{1 + \cos x - \sin x}; \text{ г) } \int_0^{\pi} \sin \frac{x}{3} \cos \frac{x}{2} dx.$$

$$6.13. \text{ а) } \int_0^{\sqrt{3}} \frac{\operatorname{arctg} x + x}{1 + x^2} dx; \text{ б) } \int_1^{\sqrt{3}} \operatorname{arctg} \frac{1}{x} dx; \text{ в) } \int_0^{\pi/2} \frac{\sin x dx}{(1 + \sin x)^2}; \text{ г) } \int_{-3}^3 x^2 \sqrt{9 - x^2} dx.$$

$$6.14. \text{ а) } \int_0^1 \frac{x}{1 + x^4} dx; \text{ б) } \int_{-\pi}^{\pi} x \sin x \cos x dx; \text{ в) } \int_{-\pi/2}^0 \frac{\cos x dx}{(1 + \cos x)^2}; \text{ г) } \int_1^2 \frac{\sqrt{x^2 - 1}}{x} dx.$$

$$6.15. \text{ а) } \int_0^2 \frac{x^3}{4 + x^2} dx; \text{ б) } \int_{\pi/4}^3 (3x - x^2) \sin 2x dx; \text{ в) } \int_0^{\pi/2} \frac{(1 + \cos x) dx}{1 + \cos x + \sin x}; \text{ г) } \int_{1/\sqrt{3}}^1 \frac{dx}{x^2 \sqrt{1 + x^2}}.$$

$$6.16. \text{ а) } \int_0^5 x \sqrt{x + 4} dx; \text{ б) } \int_{1/2}^1 \arcsin(1 - x) dx; \text{ в) } \int_0^{2\pi/3} \frac{(1 + \sin x) dx}{1 + \cos x + \sin x}; \text{ г) } \int_0^{\pi} \sin^4 \frac{x}{2} dx.$$

$$6.17. \text{ а) } \int_0^4 \frac{dx}{\sqrt{x+1}}; \text{ б) } \int_{-1/3}^{1/3} \arccos 3x dx; \text{ в) } \int_{\pi/3}^{\pi/2} \frac{\cos x dx}{1 + \sin x - \cos x}; \text{ г) } \int_0^2 \frac{dx}{(4 + x^2) \sqrt{4 + x^2}}.$$

$$6.18. \text{ а) } \int_{\pi}^{2\pi} \frac{x + \cos x}{x^2 + 2 \sin x} dx; \text{ б) } \int_{-2}^0 (x^2 - 4) \cos 4x dx; \text{ в) } \int_0^{\pi/2} \frac{\cos x dx}{1 + \cos x + \sin x}; \text{ г) } \int_{\pi/6}^{\pi/3} \frac{\sin 2x}{\cos^3 x} dx.$$

$$6.19. \text{ а) } \int_0^{\pi/4} \frac{2 \cos x + 3 \sin x}{(2 \sin x - 3 \cos x)^3} dx; \text{ б) } \int_1^e \frac{\ln^2 x}{x^2} dx; \text{ в) } \int_0^{\pi/2} \frac{\cos x dx}{1 + \sin^2 x}; \text{ г) } \int_{-1/2}^{1/2} \frac{dx}{(1 - x^2)\sqrt{1 - x^2}}.$$

$$6.20. \text{ а) } \int_0^{\pi/4} \operatorname{tg} x \ln \cos x dx; \text{ б) } \int_0^{\pi/2} (x^2 - 5x + 6) \sin 3x dx; \text{ в) } \int_0^{\pi/2} \frac{\cos x dx}{5 + \cos x}; \text{ г) } \int_{1/2}^{\sqrt{3}/2} \sqrt{1 - x^2} dx.$$

$$6.21. \text{ а) } \int_0^1 \frac{3 + \sqrt{x+2}}{x+3} dx; \text{ б) } \int_0^{2\pi} x^2 \cos 4x dx; \text{ в) } \int_0^{\pi/2} \frac{\cos x dx}{3 + 2 \cos x}; \text{ г) } \int_{\pi/4}^{\pi} 2 \sin x \cos 3x dx.$$

$$6.22. \text{ а) } \int_0^4 x^2 \sqrt{1-x} dx; \text{ б) } \int_0^{\sqrt{3}} x \operatorname{arctg} x dx; \text{ в) } \int_0^{\pi/2} \frac{(\cos x - \sin x) dx}{(1 + \sin x)^2}; \text{ г) } \int_0^{\pi/4} \sin^3 2x dx.$$

$$6.23. \text{ а) } \int_1^4 \frac{1/(2\sqrt{x}) + 1}{(\sqrt{x} + x)^2} dx; \text{ б) } \int_1^2 \ln(3x + 2) dx; \text{ в) } \int_{\pi/2}^{\pi} \frac{\sin x dx}{(1 - \cos x)^2}; \text{ г) } \int_2^4 \frac{\sqrt{x^2 - 4}}{x} dx.$$

$$6.24. \text{ а) } \int_{\pi/4}^{\pi/2} \frac{x \cos x + \sin x}{(x \sin x)^2} dx; \text{ б) } \int_0^{2\pi} (2x^2 - 4) \sin x dx; \text{ в) } \int_0^{\pi/2} \frac{(1 + \sin x) dx}{(1 - \sin x)^2}; \text{ г) } \int_0^{\pi/2} \sin^6 x dx.$$

$$6.25. \text{ а) } \int_0^1 x \sqrt[3]{2-x} dx; \text{ б) } \int_1^2 \frac{\ln(x+1)}{(x+1)^2} dx; \text{ в) } \int_0^{\pi/2} \frac{(\cos x + \sin x) dx}{(1 + \cos x)^2}; \text{ г) } \int_0^{2\pi} \sin^8 x dx.$$

$$6.26. \text{ а) } \int_1^{\sqrt{e}} \frac{dx}{x \sqrt{1 - \ln^2 x}}; \text{ б) } \int_0^3 (x^2 - 3x) \sin 2x dx; \text{ в) } \int_0^{\pi/2} \frac{\sin x dx}{1 + \cos x + \sin x}; \text{ г) } \int_{\sqrt{3}}^2 \frac{dx}{x^2 \sqrt{x^2 - 3}}.$$

$$6.27. \text{ а) } \int_1^{\sqrt{2}} \frac{x dx}{\sqrt{4 - x^2}}; \text{ б) } \int_0^{2\pi} (3 - 7x^2) \cos 2x dx; \text{ в) } \int_{\pi/6}^{\pi/3} \frac{\sin 2x dx}{\cos^3 x}; \text{ г) } \int_3^6 \frac{\sqrt{x^2 - 9}}{x^4} dx.$$

$$6.28. \text{ а) } \int_0^{\pi/4} \operatorname{ctg} x \ln \sin x dx; \text{ б) } \int_{-4}^0 (x^2 + 7x + 12) \cos x dx; \text{ в) } \int_{\pi/2}^{\pi} \frac{\cos x dx}{6 - 2 \sin x}; \text{ г) } \int_0^{\pi/2} \cos^5 x dx.$$

$$6.29. \text{ а) } \int_0^1 x^2 \sqrt{9 - x^2} dx; \text{ б) } \int_0^{2\pi} (3 - 7x^2) \sin 2x dx; \text{ в) } \int_{\pi/2}^{\pi} \frac{\sin x dx}{5 - 2 \sin x}; \text{ г) } \int_0^{\pi} \sin \frac{x}{3} \sin \frac{2x}{3} dx.$$

$$6.30. \text{ а) } \int_0^1 \frac{x^3}{x^8 + 1} dx; \text{ б) } \int_{\pi}^{2\pi} (2 + 3x) \cos 2x dx; \text{ в) } \int_{\pi/2}^{\pi} \frac{\sin x dx}{(1 + \sin x)^2}; \text{ г) } \int_{\pi/6}^{\pi/3} \cos 4x \sin 2x dx.$$

7. Вычислить несобственные интегралы или доказать их расходимость.

$$7.1. \text{ а) } \int_0^{\infty} x e^{-x^2} dx; \text{ б) } \int_1^3 \frac{dx}{\sqrt{x^2 - 6x + 9}}. \quad 7.2. \text{ а) } \int_1^{\infty} \frac{dx}{1 + x^2}; \text{ б) } \int_{1/4}^1 \frac{dx}{20x^2 - 9x + 1}.$$

$$7.3. \text{ а) } \int_{-\infty}^{\infty} \frac{2x dx}{1 + x^2}; \text{ б) } \int_0^2 \frac{dx}{x^2 - 4x + 3}. \quad 7.4. \text{ а) } \int_2^{\infty} \frac{\ln x}{x} dx; \text{ б) } \int_{\pi/2}^{\pi} \frac{\sin x dx}{\sqrt[7]{\cos^2 x}}.$$

$$7.5. \text{ а) } \int_4^{\infty} \frac{x dx}{\sqrt{x^2 - 4x + 1}}; \text{ б) } \int_1^e \frac{dx}{x \ln x}. \quad 7.6. \text{ а) } \int_1^{\infty} \frac{dx}{x^2 + x}; \text{ б) } \int_1^2 \frac{dx}{\sqrt[5]{4x - x^2 - 4}}.$$

7.7. а) $\int_1^{\infty} \frac{4dx}{x(1+\ln^2 x)}$; б) $\int_0^1 x \ln x dx$. 7.8. а) $\int_0^{\infty} e^{-\sqrt{x}} dx$; б) $\int_{-1}^1 \frac{x+1}{\sqrt[5]{x^3}} dx$.

7.9. а) $\int_{-\infty}^{\infty} \frac{dx}{x^2+2x+2}$; б) $\int_0^1 \frac{2xdx}{\sqrt{1-x^4}}$. 7.10. а) $\int_0^{\infty} \frac{xdx}{\sqrt[4]{(16+x^2)^5}}$; б) $\int_{1/3}^1 \frac{\ln(3x-1)}{3x-1} dx$.

7.11. а) $\int_0^{\infty} x \sin x dx$; б) $\int_{1/2}^1 \frac{dx}{\sqrt[9]{1-2x}}$. 7.12. а) $\int_{-1}^{\infty} \frac{xdx}{x^2+4x+5}$; б) $\int_0^{\pi/2} \frac{3\sin^3 x dx}{\sqrt{\cos x}}$.

7.13. а) $\int_0^{\infty} x^2 e^{-x^3} dx$; б) $\int_0^1 \frac{x^4 dx}{\sqrt[3]{1-x^5}}$. 7.14. а) $\int_2^{\infty} \frac{dx}{x^2+x-2}$; б) $\int_0^{\pi/6} \frac{\cos 3x}{\sqrt[6]{(1-\sin 3x)^5}} dx$.

7.15. а) $\int_1^{\infty} \frac{x^3+1}{x^4} dx$; б) $\int_1^2 \frac{xdx}{\sqrt{x-1}}$. 7.16. а) $\int_{e^2}^{\infty} \frac{dx}{x \ln \ln x}$; б) $\int_1^{3/2} \frac{dx}{\sqrt{3x-x^2-2}}$.

7.17. а) $\int_{e^2}^{\infty} \frac{dx}{x(\ln x-1)^2}$; б) $\int_{-1}^1 \frac{dx}{\sqrt{1-x^2}}$. 7.18. а) $\int_{2/\pi}^{\infty} \frac{1}{x^2} \sin \frac{1}{x} dx$; б) $\int_{-2}^2 \frac{2xdx}{x^2-4}$.

7.19. а) $\int_1^{\infty} \frac{dx}{9x^2-9x+2}$; б) $\int_0^1 \frac{xdx}{1-x^2}$. 7.20. а) $\int_0^{\infty} \frac{3-x^2}{x^2+4} dx$; б) $\int_{-1/3}^0 \frac{dx}{\sqrt[3]{1+3x}}$.

7.21. а) $\int_1^{\infty} \frac{16xdx}{16x^4-1}$; б) $\int_{3/4}^1 \frac{dx}{\sqrt[7]{3-4x}}$. 7.22. а) $\int_0^{\infty} x e^{-3x} dx$; б) $\int_1^2 \frac{xdx}{\sqrt{x-1}}$.

7.23. а) $\int_{-\infty}^0 \frac{xdx}{\sqrt{(x^2+4)^3}}$; б) $\int_0^1 x \ln x dx$. 7.24. а) $\int_0^{\infty} \frac{x^3 dx}{16x^4+1}$; б) $\int_0^{2/3} \frac{\sqrt[3]{\ln(2-3x)}}{2-3x} dx$.

7.25. а) $\int_0^{\infty} x e^x dx$; б) $\int_2^6 \frac{dx}{\sqrt[8]{(4-x)^2}}$. 7.26. а) $\int_0^{\infty} \frac{\arctg 3x}{\pi(1+9x^2)} dx$; б) $\int_0^{\pi/2} \frac{e^{\operatorname{tg} x}}{\cos^2 x} dx$.

7.27. а) $\int_0^{\infty} \frac{x^2 dx}{\sqrt[3]{(x^3+8)^4}}$; б) $\int_0^1 \frac{2xdx}{\sqrt{1-x^4}}$. 7.28. а) $\int_{-\infty}^{-1} \frac{5dx}{(x^2-4)\ln 5}$; б) $\int_1^e \frac{dx}{x\sqrt{\ln x}}$.

7.29. а) $\int_e^{\infty} \frac{dx}{x\sqrt{\ln^3 x}}$; б) $\int_{1/2}^1 \frac{dx}{\sqrt[8]{1-2x}}$. 7.30. а) $\int_{-\infty}^0 \frac{xdx}{\sqrt{(x^2+4)^3}}$; б) $\int_0^{1/e} \frac{dx}{x \ln^2 x}$.

8. Вычислить площадь фигуры, ограниченной линиями, заданными уравнениями в декартовых координатах.

8.1. $y = x^2 + x, y = 3 - x$. 8.2. $y = x^2 + 4x, y = x$. 8.3. $y = x^2, y = 2 - x$.

8.4. $y = \frac{2}{x}, y = 3 - x$. 8.5. $y = \frac{6}{x}, y + x = -5$. 8.6. $y = -\frac{2}{x}, y = 3 + x$.

8.7. $y = \sqrt{x}, y = x$. 8.8. $y = -\sqrt{x}, y = -x^2$. 8.9. $y = \sqrt{x}, y = x - 2$.

8.10. $y = \frac{2}{x^2+1}, y = x^2$. 8.11. $y = \frac{32}{x^2+4}, y = x^2$. 8.12. $y = \frac{90}{x^2+1}, y = x^2$.

8.13. $y = x^2 - 4x, y = -x^2$. 8.14. $y = x^2 - 1, y = x - x^2$. 8.15. $y = x^2, y = 8 - x^2$.

8.16. $y = x^2 - x, y = 2x + 2.$	8.17. $y = x^2 - x, y = 4x.$	8.18. $y = x^2 - 6, y = -x.$
8.19. $y = \frac{4}{x^2}, y = 5 - x^2$	8.20. $y = \frac{8}{x^3}, y = 9 - x^3$	8.21. $y = \frac{10}{x^2}, y = 8 - x^2.$
8.22. $y = \sqrt{x^3}, y = x^3$	8.23. $y = -\sqrt{x^3}, y = -x^3$	8.24. $y = \sqrt{x^3}, y = 3x.$
8.25. $y = \frac{3}{x^2 - 4}, y = x^2 - 2.$	8.26. $y = \frac{5}{x^2 - 9}, y = x^2 - 5.$	8.27. $y = \frac{3}{4 - x^2}, y = x^2.$
8.28. $y = x^2 + 8x, y = -x^2.$	8.29. $y = x^2 - 1, y = 1 - x^2.$	8.30. $y = x^2, y = 2 - x^2.$

9. Вычислить площадь фигуры, ограниченной линиями, заданными уравнениями в полярных координатах.

9.1. $r = 2\sin \varphi, r = 4\sin \varphi.$	9.2. $r = \cos 3\varphi.$	9.3. $r = \sin \varphi.;$
9.4. $r = \sin 3\varphi.$	9.5. $r = \cos 2\varphi.$	9.6. $r = 1/2 + \sin \varphi.$
9.7. $r = \sin \varphi, r = 2\sin \varphi.$	9.8. $r = 1/2 + \cos \varphi.$	9.9. $r = 4\cos 4\varphi.$
9.10. $r = 2 + \cos \varphi.$	9.11. $r = 2\sin 4\varphi.$	9.12. $r = \cos \varphi - \sin \varphi.$
9.13. $r = \sin 6\varphi.$	9.14. $r = \cos \varphi + \sin \varphi.$	9.15. $r = 2\cos 6\varphi.$
9.16. $r = 6\sin \varphi, r = 4\sin \varphi.$	9.17. $r = 4\sin^2 \varphi.$	9.18. $r = 4\sqrt{\cos 2\varphi}.$
9.19. $r = 3\sin \varphi, r = 5\sin \varphi.$	9.20. $r = 3\cos 5\varphi.$	9.21. $r = \sin 4\varphi.$
9.22. $r = \sqrt{\sin 2\varphi}.$	9.23. $r = \sin 5\varphi.$	9.24. $r = 8\cos 8\varphi.$
9.25. $r = 6\cos 6\varphi.$	9.26. $r = 8\sin 8\varphi.$	9.27. $r = 2 + 2\cos \varphi.$
9.28. $r = 1 + \sqrt{2} \cos \varphi.$	9.29. $r = 1 + \sqrt{2} \sin \varphi.$	9.30. $r = 4\sin 3\varphi.$

10. Вычислить площадь фигуры, ограниченной линиями, которые заданы параметрическим образом.

10.1. $\begin{cases} x = \sqrt{2} \cos t, \\ y = 4\sqrt{2} \sin t, \\ y = 4 (y \geq 4). \end{cases}$	10.2. $\begin{cases} x = 4\sqrt{2} \cos^3 t, \\ y = \sqrt{2} \sin^3 t, \\ x = 2 (x \geq 2). \end{cases}$	10.3. $\begin{cases} x = 4(t - \sin t), \\ y = 4(1 - \cos t), \\ y = 6 (0 < x < 8\pi, y \geq 6). \end{cases}$
10.4. $\begin{cases} x = 8\cos^3 t, \\ y = 8\sin^3 t, \\ x = 1 (x \geq 1). \end{cases}$	10.5. $\begin{cases} x = 16\cos^3 t, \\ y = \sin^3 t, \\ x = 6\sqrt{3} (x \geq 6\sqrt{3}). \end{cases}$	10.6. $\begin{cases} x = 2(t - \sin t), \\ y = 2(1 - \cos t), \\ y = 3 (0 < x < 4\pi, y \geq 3). \end{cases}$
10.7. $\begin{cases} x = 3\cos t, \\ y = 8\sin t, \\ y = 4 (y \geq 4). \end{cases}$	10.8. $\begin{cases} x = 32\cos^3 t, \\ y = 3\sin^3 t, \\ x = 12\sqrt{3} (x \geq 12\sqrt{3}). \end{cases}$	10.9. $\begin{cases} x = 6(t - \sin t), \\ y = 6(1 - \cos t), \\ y = 6 (0 < x < 12\pi, y \geq 6). \end{cases}$

$$10.10. \begin{cases} x = \sqrt{2} \cos t, \\ y = 2\sqrt{2} \sin t, \\ y = 2 \ (y \geq 2). \end{cases} \quad 10.11. \begin{cases} x = 6 \cos t, \\ y = 4 \sin t, \\ y = 2\sqrt{3} \ (y \geq 2\sqrt{3}). \end{cases} \quad 10.12. \begin{cases} x = 3(t - \sin t), \\ y = 3(1 - \cos t), \\ y = 3 \ (0 < x < 6\pi, y \geq 3). \end{cases}$$

$$10.13. \begin{cases} x = 2\sqrt{2} \cos^3 t, \\ y = \sqrt{2} \sin^3 t, \\ x = 1 \ (x \geq 1). \end{cases} \quad 10.14. \begin{cases} x = 3 \cos t, \\ y = 8 \sin t, \\ y = 4\sqrt{3} \ (y \geq 4\sqrt{3}). \end{cases} \quad 10.15. \begin{cases} x = 10(t - \sin t), \\ y = 10(1 - \cos t), \\ y = 5 \ (0 < x < 20\pi, y \geq 5). \end{cases}$$

$$10.16. \begin{cases} x = 9 \cos t, \\ y = 4 \sin t, \\ y = 2 \ (y \geq 2). \end{cases} \quad 10.17. \begin{cases} x = 2\sqrt{2} \cos t, \\ y = 5\sqrt{2} \sin t, \\ y = 5 \ (y \geq 5). \end{cases} \quad 10.18. \begin{cases} x = 8(t - \sin t), \\ y = 8(1 - \cos t), \\ y = 2 \ (0 < x < 16\pi, y \geq 2). \end{cases}$$

$$10.19. \begin{cases} x = 2\sqrt{2} \cos t, \\ y = 3\sqrt{2} \sin t, \\ y = 3 \ (y \geq 3). \end{cases} \quad 10.20. \begin{cases} x = 6 \cos t, \\ y = 2 \sin t, \\ y = \sqrt{3} \ (y \geq \sqrt{3}). \end{cases} \quad 10.21. \begin{cases} x = 6(t - \sin t), \\ y = 6(1 - \cos t), \\ y = 9 \ (0 < x < 12\pi, y \geq 9). \end{cases}$$

$$10.22. \begin{cases} x = 2 \cos t, \\ y = 6 \sin t, \\ y = 3 \ (y \geq 3). \end{cases} \quad 10.23. \begin{cases} x = 24 \cos^3 t, \\ y = 2 \sin^3 t, \\ x = 9\sqrt{3} \ (x \geq 9\sqrt{3}). \end{cases} \quad 10.24. \begin{cases} x = 2(t - \sin t), \\ y = 2(1 - \cos t), \\ y = 2 \ (0 < x < 4\pi, y \geq 2). \end{cases}$$

$$10.25. \begin{cases} x = 16 \cos^3 t, \\ y = 2 \sin^3 t, \\ x = 2 \ (x \geq 2). \end{cases} \quad 10.26. \begin{cases} x = 4\sqrt{2} \cos^3 t, \\ y = 2\sqrt{2} \sin^3 t, \\ x = 2 \ (x \geq 2). \end{cases} \quad 10.27. \begin{cases} x = t - \sin t, \\ y = 1 - \cos t, \\ y = 1 \ (0 < x < 2\pi, y \geq 1). \end{cases}$$

$$10.28. \begin{cases} x = 32 \cos^3 t, \\ y = \sin^3 t, \\ x = 4 \ (x \geq 4). \end{cases} \quad 10.29. \begin{cases} x = 8 \cos^3 t, \\ y = 4 \sin^3 t, \\ x = 3\sqrt{3} \ (x \geq 3\sqrt{3}). \end{cases} \quad 10.30. \begin{cases} x = 4(t - \sin t), \\ y = 4(1 - \cos t), \\ y = 4 \ (0 < x < 8\pi, y \geq 4). \end{cases}$$

11. Вычислить длину дуги кривой, заданной уравнением в декартовых координатах.

$$10.1. y = e^x + e, \ln \sqrt{3} \leq x \leq \ln \sqrt{15}. \quad 10.2. y = 2 + \arcsin \sqrt{x} + \sqrt{1-x^2}, 1/4 \leq x \leq 1.$$

$$10.3. y = e^x + 25, \ln \sqrt{8} \leq x \leq \ln \sqrt{24}. \quad 10.4. y = -\arccos x + \sqrt{1-x^2} + 1, 0 \leq x \leq 9/16.$$

$$10.5. y = \ln x, \sqrt{3} \leq x \leq \sqrt{15}. \quad 10.6. y = \arccos \sqrt{x} - \sqrt{1-x^2} + 4, 0 \leq x \leq 1/2.$$

$$10.7. y = \frac{x^2}{4} - \frac{\ln x}{2}, 1 \leq x \leq 2. \quad 10.8. y = \sqrt{1-x^2} - \arccos \sqrt{x} + 5, 1/9 \leq x \leq 1.$$

$$10.9. y = \frac{e^x + e^{-x}}{2} + 3, 0 \leq x \leq 2.$$

$$10.11. y = \ln \frac{5}{2x}, \sqrt{3} \leq x \leq \sqrt{8}.$$

$$10.13. y = -\ln \cos x, 0 \leq x \leq \pi/6.$$

$$10.15. y = \ln(x^2 - 1), 2 \leq x \leq 3.$$

$$10.17. y = \ln(1 - x^2), 0 \leq x \leq 1/4.$$

$$10.19. y = 2 + \operatorname{ch} x, 0 \leq x \leq 1.$$

$$10.21. y = 2 - e^x, \ln \sqrt{3} \leq x \leq \ln \sqrt{8}.$$

$$10.23. y = 1 - \ln(x^2 - 1), 3 \leq x \leq 4.$$

$$10.25. y = \ln 7 - \ln x, \sqrt{3} \leq x \leq \sqrt{8}.$$

$$10.27. y = \ln \cos x + 2, 0 \leq x \leq \pi/6.$$

$$10.29. y = (e^{2x} + e^{-2x} + 3)/4, 0 \leq x \leq 2.$$

$$10.10. y = e^x + 6, \ln \sqrt{8} \leq x \leq \ln \sqrt{15}.$$

$$10.12. y = 1 + \arcsin x - \sqrt{1 - x^2}, 0 \leq x \leq 3/4.$$

$$10.14. y = \sqrt{1 - x^2} + \arccos x, 0 \leq x \leq 8/9.$$

$$10.16. y = \arcsin x - \sqrt{1 - x^2}, 0 \leq x \leq 15/16.$$

$$10.18. y = -\arccos \sqrt{x} + \sqrt{1 - x^2}, 0 \leq x \leq 1/4.$$

$$10.20. y = 1 - \ln \cos x, 0 \leq x \leq \pi/6.$$

$$10.22. y = 1 - \ln \sin x, \pi/3 \leq x \leq \pi/2.$$

$$10.24. y = \ln \sin x, \pi/3 \leq x \leq \pi/2.$$

$$10.26. y = \operatorname{ch} x + 3, 0 \leq x \leq 1.$$

$$10.28. y = e^x + 13, \ln \sqrt{15} \leq x \leq \ln \sqrt{24}.$$

$$10.30. y = (1 - e^x - e^{-x})/2, 0 \leq x \leq 3.$$

11. Вычислить длину дуги кривой, заданной уравнением в полярных координатах.

$$11.1. r = 3e^{3\varphi/4}, -\pi/2 \leq \varphi \leq \pi/2.$$

$$11.3. r = \sqrt{2}e^\varphi, -\pi/2 \leq \varphi \leq \pi/2.$$

$$11.5. r = 6e^{12\varphi/5}, -\pi/2 \leq \varphi \leq \pi/2.$$

$$11.7. r = 4e^{4\varphi/3}, 0 \leq \varphi \leq \pi/3.$$

$$11.9. r = 5e^{5\varphi/12}, 0 \leq \varphi \leq \pi/3.$$

$$11.11. r = 2\varphi, 0 \leq \varphi \leq 4/3.$$

$$11.13. r = 2\varphi, 0 \leq \varphi \leq 12/5.$$

$$11.15. r = 3\varphi, 0 \leq \varphi \leq 4/3.$$

$$11.17. r = 2\cos \varphi, 0 \leq \varphi \leq \pi/6.$$

$$11.19. r = 2\varphi, 0 \leq \varphi \leq 3/4.$$

$$11.21. r = 2\varphi, 0 \leq \varphi \leq 5/12.$$

$$11.23. r = 4\varphi, 0 \leq \varphi \leq 3/4.$$

$$11.25. r = 5\varphi, 0 \leq \varphi \leq 12/5.$$

$$11.27. r = 8\cos \varphi, 0 \leq \varphi \leq \pi/4.$$

$$11.29. r = 2\sin \varphi, 0 \leq \varphi \leq \pi/6.$$

$$11.2. r = 2e^{4\varphi/3}, -\pi/2 \leq \varphi \leq \pi/2.$$

$$11.4. r = 5e^{5\varphi/12}, -\pi/2 \leq \varphi \leq \pi/2.$$

$$11.6. r = 3e^{3\varphi/12}, 0 \leq \varphi \leq \pi/3.$$

$$11.8. r = \sqrt{2}e^\varphi, 0 \leq \varphi \leq \pi/3.$$

$$11.10. r = 12e^{12\varphi/5}, 0 \leq \varphi \leq \pi/3.$$

$$11.12. r = 2(1 - \cos \varphi), -\pi \leq \varphi \leq -\pi/2.$$

$$11.14. r = 4(1 - \sin \varphi), 0 \leq \varphi \leq \pi/6.$$

$$11.16. r = 6(1 + \sin \varphi), -\pi/2 \leq \varphi \leq 0.$$

$$11.18. r = 8(1 - \cos \varphi), -2\pi/3 \leq \varphi \leq 0.$$

$$11.20. r = 1 - \sin \varphi, -\pi/2 \leq \varphi \leq -\pi/6.$$

$$11.22. r = 3(1 + \sin \varphi), -\pi/6 \leq \varphi \leq 0.$$

$$11.24. r = 5(1 - \cos \varphi), -\pi/3 \leq \varphi \leq 0.$$

$$11.26. r = 7(1 - \sin \varphi), -\pi/6 \leq \varphi \leq \pi/6.$$

$$11.28. r = 6\cos \varphi, 0 \leq \varphi \leq \pi/3.$$

$$11.30. r = 8\sin \varphi, 0 \leq \varphi \leq \pi/4.$$

12. Вычислить длину дуги кривой, заданной уравнениями параметрическим образом.

$$12.1. \begin{cases} x = 5(t - \sin t), \\ y = 5(1 - \cos t), \\ 0 \leq t \leq \pi. \end{cases} \quad 12.2. \begin{cases} x = 3(2 \cos t - \cos 2t), \\ y = 3(2 \sin t - \sin 2t), \\ 0 \leq t \leq 2\pi. \end{cases} \quad 12.3. \begin{cases} x = 4(\cos t + t \sin t), \\ y = 4(\sin t - t \cos t), \\ 0 \leq t \leq 2\pi. \end{cases}$$

$$12.4. \begin{cases} x = 4 \cos^3 t, \\ y = 4 \sin^3 t, \\ 0 \leq t \leq \frac{\pi}{4}. \end{cases} \quad 12.5. \begin{cases} x = (1,5 \cdot t^2 - 3) \sin t + 3t \cos t, \\ y = (3 - 1,5 \cdot t^2) \cos t + 3t \sin t, \\ 0 \leq t \leq \pi/2. \end{cases} \quad 12.6. \begin{cases} x = e^t (\cos t + \sin t), \\ y = e^t (\cos t - \sin t), \\ 0 \leq t \leq \pi. \end{cases}$$

$$12.7. \begin{cases} x = 3(t + \sin t), \\ y = 3(1 - \cos t), \\ \pi \leq t \leq 2\pi. \end{cases} \quad 12.8. \begin{cases} x = \frac{1}{2} \cos t - \frac{1}{4} \cos 2t, \\ y = \frac{1}{2} \sin t - \frac{1}{4} \sin 2t, \\ \frac{\pi}{2} \leq t \leq \frac{2\pi}{3}. \end{cases} \quad 12.9. \begin{cases} x = 3(\sin t - t \cos t), \\ y = 3(\cos t + t \sin t), \\ 0 \leq t \leq \frac{\pi}{3}. \end{cases}$$

$$12.10. \begin{cases} x = 6 \cos^3 t, \\ y = 6 \sin^3 t, \\ 0 \leq t \leq \pi/3. \end{cases} \quad 12.11. \begin{cases} x = e^t (\cos t - \sin t), \\ y = e^t (\cos t + \sin t), \\ \pi/2 \leq t \leq \pi. \end{cases} \quad 12.12. \begin{cases} x = e^{2t} (\cos 2t + \sin 2t), \\ y = e^{2t} (\cos 2t - \sin 2t), \\ \pi/6 \leq t \leq \pi/4. \end{cases}$$

$$12.13. \begin{cases} x = (t^2 - 2) \sin t + 2t \cos t, \\ y = (2 - t^2) \cos t + 2t \sin t, \\ 0 \leq t \leq \pi/3. \end{cases} \quad 12.14. \begin{cases} x = 10 \cos^3 2t, \\ y = 10 \sin^3 2t, \\ 0 \leq t \leq \pi/4. \end{cases} \quad 12.15. \begin{cases} x = 2 \cos^2 t, \\ y = 2 \sin^2 t, \\ 0 \leq t \leq \pi/4. \end{cases}$$

$$12.16. \begin{cases} x = 8 \cos^3 3t, \\ y = 8 \sin^3 3t, \\ 0 \leq t \leq \pi/18. \end{cases} \quad 12.17. \begin{cases} x = \cos 2t + 2t \sin 2t, \\ y = \sin 2t - 2t \cos 2t, \\ 0 \leq t \leq \pi/4. \end{cases} \quad 12.18. \begin{cases} x = 3,5 \cdot (2 \cos t + \cos 2t), \\ y = 3,5 \cdot (2 \sin t + \sin 2t), \\ 0 \leq t \leq \pi/2. \end{cases}$$

$$12.19. \begin{cases} x = e^t (\cos 3t - \sin 3t), \\ y = e^t (\cos 3t + \sin 3t), \\ 0 \leq t \leq 2\pi. \end{cases} \quad 12.20. \begin{cases} x = 4(t - \sin t), \\ y = 4(1 + \cos t), \\ \pi/2 \leq t \leq 2\pi/3. \end{cases} \quad 12.21. \begin{cases} x = 2 \cdot (2 \cos t - \cos 2t), \\ y = 2 \cdot (2 \sin t + \sin 2t), \\ 0 \leq t \leq \pi/3. \end{cases}$$

$$12.22. \begin{cases} x = \left(\frac{t^2}{2} - 1\right) \sin t + t \cos t, \\ y = \left(1 - \frac{t^2}{2}\right) \cos t + t \sin t, \\ 0 \leq t \leq 2\pi. \end{cases} \quad 12.23. \begin{cases} x = 3(t + \sin t), \\ y = 3(1 + \cos t), \\ \frac{\pi}{2} \leq t \leq \frac{3\pi}{4}. \end{cases} \quad 12.24. \begin{cases} x = 2,5 \cdot (1 - \cos t), \\ y = 2,5 \cdot (t - \sin t), \\ \frac{\pi}{4} \leq t \leq \frac{5\pi}{6}. \end{cases}$$

$$\begin{aligned}
12.25. & \begin{cases} x = \cos \frac{t}{2} + \frac{t}{2} \sin \frac{t}{2}, \\ y = \sin \frac{t}{2} - \frac{t}{2} \cos \frac{t}{2}, \\ 0 \leq t \leq \pi / 4. \end{cases} & 12.26. & \begin{cases} x = 5 \cos^2 4t, \\ y = 5 \sin^2 4t, \\ \frac{\pi}{24} \leq t \leq \frac{\pi}{12} \end{cases} & 12.27. & \begin{cases} x = e^{\frac{t}{2}} \left(\cos \frac{t}{2} - \sin \frac{t}{2} \right), \\ y = e^{\frac{t}{2}} \left(\cos \frac{t}{2} + \sin \frac{t}{2} \right), \\ 0 \leq t \leq 3\pi. \end{cases} \\
12.28. & \begin{cases} x = 7(1 + \cos t), \\ y = 7(t + \sin t), \\ \pi / 2 \leq t \leq 3\pi / 2. \end{cases} & 12.29. & \begin{cases} x = 4 \cdot (2 \cos t + \cos 2t), \\ y = 4 \cdot (2 \sin t - \sin 2t), \\ 0 \leq t \leq \pi. \end{cases} & 12.30. & \begin{cases} x = \cos 5t + 5t \sin 5t, \\ y = 5 \sin t - 5t \cos 5t, \\ 0 \leq t \leq \pi^2 / 4. \end{cases}
\end{aligned}$$

13. Вычислить объем тела, образованного вращением фигуры, ограниченной графиками функций. В вариантах 1-15 ось вращения Ox , в вариантах 16-30 ось вращения Oy .

13.1. $xy = 4, 2x + y - 6 = 0$.

13.2. $y^2 = (x + 4)^3, x = 0$.

13.3. $y = x^3, y = \sqrt{x}$.

13.4. $y = x^2, y = 1, x = 2$.

13.5. $y = 1 - x^2, x = 0, x = \sqrt{y - 2}$.

13.6. $x^2 + (y - 2)^2 = 1$.

13.7. $y = x^2, y^2 - x = 0$.

13.8. $y = e^{1-x}, y = 0, x = 0, x = 1$.

13.9. $y = 2x - x^2, y = -x + 2, x = 0$.

13.10. $y = xe^x, y = 0, x = 1$.

13.11. $x = \sqrt[3]{y - 2}, x = 1, y = 1$.

13.12. $y = -x^2 + 5x - 6, y = 0$.

13.13. $2x - x^2 - y = 0, 2x^2 - 4x + y = 0$.

13.14. $y = 3 \sin x, y = \sin x, 0 \leq x \leq \pi$.

13.15. $y = 5 \cos x, y = \cos x, x = 0, x \geq 0$.

13.16. $y = (x - 1)^2, x = 0, x = 2, y = 0$.

13.17. $y = x^3, y = x$.

13.18. $y = x^2 - 2x + 1, x = 2, y = 0$.

13.19. $y = x^3, y = x^2$.

13.20. $y^2 = x - 2, y = 0, y = x^3, y = 1$.

13.21. $y = (x - 1)^2, y = 1$.

13.22. $y = \ln x, x = 2, y = 0$.

13.23. $y = \sqrt{x - 1}, y = 0, y = 1, x = 0.5$.

13.24. $y = x^2 + 1, y = x, x = 0, x = 1$.

13.25. $y = x^2, x = 2, y = 0$.

13.26. $x^2/9 + y^2/4 = 1$.

13.27. $y = x^2, 8x = y^2$.

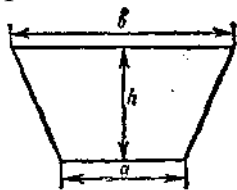
13.28. $y = x^3, x = 0, y = 8$.

13.29. $y = 2 - x^2/2, x + y = 2$.

13.30. $y^2 = 4 - x, x = 0$.

14. Используя определенный интеграл решить следующие задачи.

Варианты 1-10:



Вычислить силу, с которой вода давит на плотину, сечение которой имеет форму равнобоковой трапеции (см. рис.). Плотность воды $\rho = 1000 \text{ кг/м}^3$, ускорение свободного падения g положить равным 10 м/с^2 .

Указание. Давление на глубине x равно $\rho g x$.

14.1. $a = 4.5 \text{ м}, b = 6.6 \text{ м}, h = 3.0 \text{ м}.$

14.2. $a = 4.8 \text{ м}, b = 7.2 \text{ м}, h = 3.0 \text{ м}.$

14.3. $a = 5.1 \text{ м}, b = 7.8 \text{ м}, h = 3.0 \text{ м}.$

14.4. $a = 5.4 \text{ м}, b = 8.4 \text{ м}, h = 3.0 \text{ м}.$

14.5. $a = 5.7 \text{ м}, b = 9.0 \text{ м}, h = 4.0 \text{ м}.$

14.6. $a = 6.0 \text{ м}, b = 9.6 \text{ м}, h = 4.0 \text{ м}.$

14.7. $a = 6.3 \text{ м}, b = 10.2 \text{ м}, h = 4.0 \text{ м}.$

14.8. $a = 6.6 \text{ м}, b = 10.8 \text{ м}, h = 4.0 \text{ м}.$

14.9. $a = 6.9 \text{ м}, b = 11.4 \text{ м}, h = 5.0 \text{ м}.$

14.10. $a = 7.2 \text{ м}, b = 12.0 \text{ м}, h = 5.0 \text{ м}.$

Варианты 11-20: Определить работу (в джоулях), совершаемую при подъеме спутника с поверхности Земли на высоту H км. Масса спутника равна m т., радиус Земли $R_z = 6380 \text{ км}$. Ускорение свободного падения g у поверхности Земли положить равным 10 м/с^2 .

14.11. $m = 7.0 \text{ т}, H = 200 \text{ км}.$

14.12. $m = 7.0 \text{ т}, H = 250 \text{ км}.$

14.13. $m = 6.0 \text{ т}, H = 300 \text{ км}.$

14.14. $m = 6.0 \text{ т}, H = 350 \text{ км}.$

14.15. $m = 5.0 \text{ т}, H = 400 \text{ км}.$

14.16. $m = 5.0 \text{ т}, H = 450 \text{ км}.$

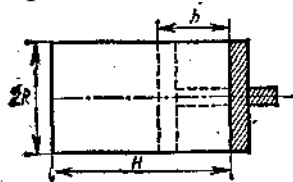
14.17. $m = 4.0 \text{ т}, H = 500 \text{ км}.$

14.18. $m = 4.0 \text{ т}, H = 550 \text{ км}.$

14.19. $m = 3.0 \text{ т}, H = 600 \text{ км}.$

14.20. $m = 3.0 \text{ т}, H = 650 \text{ км}.$

Варианты 21-30:



Цилиндр наполнен газом под атмосферным давлением (103.3 кПа). Считая газ идеальным, определить работу (в джоулях) при изотермическом сжатии газа поршнем, переместившимся внутрь цилиндра на h м (см. рис.).

Указание. Уравнение состояния газа $pV = \text{const}$, где p — давление, V — объем.

14.21. $H = 0.4 \text{ м}, h = 0.35 \text{ м}, R = 0.1 \text{ м}.$

14.22. $H = 0.4 \text{ м}, h = 0.3 \text{ м}, R = 0.1 \text{ м}.$

14.23. $H = 0.4 \text{ м}, h = 0.2 \text{ м}, R = 0.1 \text{ м}.$

14.24. $H = 0.8 \text{ м}, h = 0.7 \text{ м}, R = 0.2 \text{ м}.$

14.25. $H = 0.8 \text{ м}, h = 0.6 \text{ м}, R = 0.2 \text{ м}.$

14.26. $H = 0.8 \text{ м}, h = 0.4 \text{ м}, R = 0.2 \text{ м}.$

14.27. $H = 1.6 \text{ м}, h = 1.4 \text{ м}, R = 0.3 \text{ м}.$

14.28. $H = 1.6 \text{ м}, h = 1.2 \text{ м}, R = 0.3 \text{ м}.$

14.29. $H = 1.6 \text{ м}, h = 0.8 \text{ м}, R = 0.3 \text{ м}.$

14.30. $H = 2.0 \text{ м}, h = 1.5 \text{ м}, R = 0.4 \text{ м}.$

РАЗДЕЛ 3. ДИФФЕРЕНЦИАЛЬНЫЕ УРАВНЕНИЯ И СИСТЕМЫ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ

15. Найти общее решение или общий интеграл дифференциального уравнения. При решении задачи необходимо указать тип уравнения.

- | | |
|--|--|
| 15.1. $(xy^2 - x)dx + (yx^2 - y)dy = 0.$ | 15.2. $(xy^2 + x)dx + (yx^2 + y)dy = 0.$ |
| 15.3. $xy^2 dx + yx^2 dy + xdx + 4ydy = 0.$ | 15.4. $(1 + x^2)dy = (9 + y^2)dx.$ |
| 15.5. $(xy^2 + 3y^2)dx + (x^2 - x^2 y)dy = 0.$ | 15.6. $(x + 2)^3 dy - (y - 3)^2 dx = 0.$ |
| 15.7. $(x - 5)^4 dy - (y + 1)^3 dx = 0.$ | 15.8. $\sin 3y \cos x dy = \cos 3y \sin x dx.$ |
| 15.9. $\frac{dx}{\cos^2 x \cos y} + \operatorname{ctg} x \sin y dy = 0.$ | 15.10. $e^{2x+5y} dy - x dx = 0.$ |
| 15.11. $y' \sqrt{1 - x^2} - \cos^2 y = 0.$ | 15.12. $3^{y^2 - x^2} = \frac{y \cdot y'}{x}.$ |
| 15.13. $y' - \frac{\cos(2x + y)}{\cos^3 y} = \frac{\cos(2x - y)}{\cos^3 y}.$ | 15.14. $(1 + e^{3y})x dx = e^{3y} dy.$ |
| 15.15. $y' + \sin(x + y) = \sin(x - y).$ | 15.16. $\sin x \cdot y' = y \cdot \cos x + 2 \cos x.$ |
| 15.17. $e^x \cdot \operatorname{tg} y dx = \frac{(1 - e^x)}{\cos^2 y} dy.$ | 15.18. $\frac{\cos y}{\sqrt{1 + x^2}} dx = 2 dy + \cos y dy.$ |
| 15.19. $1 + (1 + y') \cdot e^y = 0.$ | 15.20. $e^x \cdot \sin y dx + \operatorname{tg} y dy = 0.$ |
| 15.21. $3^{x+2y} + 5^{x-2y} \cdot y' = 0.$ | 15.22. $y' \cdot (\sqrt{x \cdot y} + \sqrt{x}) - y = 0.$ |
| 15.23. $\sqrt{9 - y^2} dx + y \cdot \sqrt{4 - x^2} dy = 0.$ | 15.24. $\sec^2 x \cdot \operatorname{tg} y + y' \sec^2 y \cdot \operatorname{tg} x = 0.$ |
| 15.25. $x \cdot (y^6 + 1) dx + y^2 \cdot (x^4 + 1) dy = 0.$ | 15.26. $y \cdot \ln^3 y + y' \cdot \sqrt{x + 1} = 0.$ |
| 15.27. $dy + \sin(x + y) dx = \sin(x - y) dx.$ | 15.28. $x \sqrt{4 + y^2} dx + y \sqrt{9 + x^2} dy = 0.$ |
| 15.29. $\frac{dy}{dx} + \cos(x + 2y) = \cos(x - 2y).$ | 15.30. $\ln(\cos y) dx + x \cdot \operatorname{tg} y dy = 0.$ |

16. Найти общее решение или общий интеграл дифференциального уравнения. При решении задачи необходимо указать тип уравнения.

- | | |
|--|---|
| 16.1. $(x^2 - y^2) dy = 2 \cdot x \cdot y dx.$ | 16.2. $y \cdot (\ln y - \ln x) dx - x dy = 0.$ |
| 16.3. $(x \cdot y' - y) \cdot \operatorname{arctg} \frac{y}{x} = x.$ | 16.4. $\frac{x}{y} dy = \frac{x}{y} \cdot e^{\frac{y}{x}} dx + dx.$ |
| 16.5. $(y + x) dy = (y - x) dx.$ | 16.6. $x dy = y \cdot (1 - \ln x + \ln y) dx.$ |
| 16.7. $x \cdot y' = x \cdot \cos \frac{y}{x} + y.$ | 16.8. $x dy = \left(y + x \cdot \sin \frac{y}{x} \right) dx.$ |
| 16.9. $(x^2 + 2 \cdot x \cdot y) dx + x \cdot y dy = 0.$ | 16.10. $y = x \cdot (y' - \sqrt[3]{e^y}).$ |

16.11. $x \cdot y + y^2 = (2 \cdot x^2 + x \cdot y) \cdot y'.$	16.12. $(2 \cdot \sqrt{x \cdot y} - y) dx + x dy = 0.$
16.13. $x \cdot y' + y \cdot \left(\ln \frac{y}{x} - 1 \right) = 0.$	16.14. $(x^2 + y^2) dx + 2 \cdot x \cdot y dy = 0.$
16.15. $(y^2 - 2 \cdot x \cdot y) dx - x^2 dy = 0.$	16.16. $(x^2 - y^2) dx + 2 \cdot x \cdot y dy = 0.$
16.17. $(x + 2 \cdot y) dx + x dy = 0.$	16.18. $(2 \cdot x - y) dx + (x + y) dy = 0.$
16.19. $2 \cdot x^3 \cdot y' = y \cdot (2 \cdot x^2 - y^2).$	16.20. $y' = \frac{x}{y} + \frac{y}{x}.$
16.21. $x dy = \sqrt{x^2 - y^2} dx + y dx.$	16.22. $x dy = y dx + x \cdot \cos^2 \frac{y}{x} dx.$
16.23. $x \cdot y' = y + 2 \cdot \sqrt{y^2 + 9 \cdot x^2}.$	16.24. $x^2 dy = x \cdot y dx + y^2 \cdot e^{-x/y} dx.$
16.25. $\left(x - y \cos \frac{y}{x} \right) dx + x \cos \frac{y}{x} dy = 0.$	16.26. $2 \cdot x^2 dy = (x^2 + y^2) dx.$
16.27. $x dy - y dx = y dy.$	16.28. $(2\sqrt{x \cdot y} - y) dx + x dy = 0.$
16.29. $(y + \sqrt{x \cdot y}) dx = x dy.$	16.30. $\sqrt{y^2 + x^2} dx = y dx - x dy.$

17. Найти общий интеграл дифференциального уравнения.

17.1. $y' = \frac{2x - 3y + 1}{4x + y - 5}.$	17.2. $y' = \frac{6x - 5y + 1}{2x - y - 1}.$	17.3. $y' = \frac{4x + 3y - 7}{2x - 5y + 3}.$
17.4. $y' = \frac{7x - 12y + 5}{3x + 2y - 5}.$	17.5. $y' = \frac{4x - 5y + 1}{3x + y - 4}.$	17.6. $y' = \frac{x + 3y - 4}{5x + y - 6}.$
17.7. $y' = \frac{4x + 2y - 6}{3x + y - 4}.$	17.8. $y' = \frac{7x - 3y - 4}{4x - 9y + 5}.$	17.9. $y' = \frac{3x - 3y}{2x + 2y - 4}.$
17.10. $y' = \frac{x - y}{2x + y - 3}.$	17.11. $y' = \frac{6x + y - 7}{8x + y - 9}.$	17.12. $y' = \frac{4x - y - 3}{4x + 3y - 7}.$
17.13. $y' = \frac{4x - 5y + 1}{2x + 7y - 9}.$	17.14. $y' = \frac{x - y}{3x + 2y - 5}.$	17.15. $y' = \frac{2x + y - 3}{x - y}.$
17.16. $y' = \frac{x - 3y + 2}{x + y - 2}.$	17.17. $y' = \frac{x - 5y + 4}{9x - 4y - 5}.$	17.18. $y' = \frac{6x + y - 7}{3x + y - 4}.$
17.19. $y' = \frac{6x - 2y - 4}{3x + y - 4}.$	17.20. $y' = \frac{3x + 3y - 6}{4x + 5y - 9}.$	17.21. $y' = \frac{5x - 6y + 1}{7x + 2y - 9}.$
17.22. $y' = \frac{3x - 2y + 1}{5x + 2y - 7}.$	17.23. $y' = \frac{x - 7y + 8}{2x - y - 1}.$	17.24. $y' = \frac{x + 8y - 9}{4x - y - 3}.$
17.25. $y' = \frac{2x + y - 3}{x + 6y - 7}.$	17.26. $y' = \frac{6x - 5y - 1}{2x + 5y - 7}.$	17.27. $y' = \frac{2x - y - 1}{4x - y - 3}.$
17.28. $y' = \frac{6x - y - 5}{4x - y - 3}.$	17.29. $y' = \frac{5x - y - 4}{7x + y - 8}.$	17.30. $y' = \frac{4x + 3y - 7}{2x + y - 3}.$

18. Найти частное решение дифференциального уравнения, которое удовлетворяет начальному условию $y(x_0) = y_0$. При решении задачи необходимо указать тип уравнения.

18.1. $(x+1)y' + y = x^2$,
 $y(0) = 5$.

18.3. $y' \cos x = y \sin x + \cos^2 x$,
 $y(3\pi/2) = -1/4$.

18.5. $y' + \frac{x \cdot y}{1-x^2} = (x + \arcsin x)$,
 $y(0) = 0$.

18.7. $x \cdot y' + y - 3 \cdot \sin x = 0$,
 $y(\pi/2) = 2/\pi$.

18.9. $y' + y \cdot \cos x = e^{-\sin x}$,
 $y(0) = 0$.

18.11. $(x+1) \cdot y' + y = x^3 + x^2$,
 $y(0) = 0$.

18.13. $x \cdot y' + y = \sin x$,
 $y(\pi/2) = 2/\pi$.

18.15. $(1-x^2) \cdot y' + x \cdot y = 1$,
 $y(0) = 1$.

18.17. $x^2 \cdot y' = 2 \cdot x \cdot y + 3$,
 $y(1) = -1$.

18.19. $y' - 3 \cdot x^2 \cdot y - x^2 \cdot e^{x^3} = 0$,
 $y(0) = 0$.

18.21. $y' - y \cdot \operatorname{tg} x = \sec x$,
 $y(0) = 1$.

18.23. $\sin 2x \, dy = 2 \cdot (y + \cos x) \, dx$,
 $y(\pi/4) = -\sqrt{2}$.

18.25. $x \cdot (x-1) \cdot y' + y = x^2 \cdot (2x-1)$,
 $y(2) = 4$.

18.27. $y' - 2 \cdot y \cdot \operatorname{ctg} x = \sin^3 x$,
 $y(\pi/2) = 0$.

18.29. $y' - y \cdot \cos x = -\sin 2x$,
 $y(0) = 3$.

18.2. $y' \sin x - y \cos x = \sin^2 x$,
 $y(\pi/2) = \pi/2$.

18.4. $y' \cos^2 x + y = \operatorname{tg} x$,
 $y(0) = -1$.

18.6. $(1+x^2)y' - 2xy = 3x^2(1+x^2)^2$,
 $y(0) = 1$.

18.8. $y' - y \cdot \operatorname{tg} x = 4 \cdot x^3 \cdot \sec x$,
 $y(0) = 0$.

18.10. $xy' + y - 2x^2 = 0$,
 $y(1) = 1$.

18.12. $x \cdot y' - 2 \cdot y + x^2 = 0$,
 $y(1) = 0$.

18.14. $(x^2-1) \cdot y' - x \cdot y = x^3 - x$,
 $y(\sqrt{2}) = 1$.

18.16. $y' \cdot \operatorname{ctg} x - y = 2 \cos^2 x \cdot \operatorname{ctg} x$,
 $y(0) = 0$.

18.18. $y' - 2 \cdot x \cdot y = x \cdot e^{-x^2}$,
 $y(0) = 0$.

18.20. $x \cdot y' + y = \ln x + 1$,
 $y(1) = 0$.

18.22. $y' \cdot \sin x - y \cdot \cos x = -\frac{\sin^2 x}{x^2}$,
 $y(\pi/2) = 2/\pi$.

18.24. $y' = 2 \cdot y \cdot \sin^2 x + 2 \cdot x \cdot e^{x - \frac{1}{2}x^2}$,
 $y(0) = 0$.

18.26. $x \cdot y' + y - e^x = 0$,
 $y(1) = e$.

18.28. $y' + 2 \cdot y \cdot \operatorname{tg} x = x \cdot \cos^3 x$,
 $y(0) = 1$.

18.30. $x \cdot y' - y + \ln x = 0$,
 $y(1) = 2$.

19. Решить задачу Коши, при заданном начальном условии $y(3) = m$, где число m – номер варианта. При решении задания необходимо указать тип уравнения.

$$\begin{array}{lll}
 19.1. & y^2 y' + \frac{y^3}{x} = \frac{1}{x^2}. & 19.2. & y' + 4x^3 y = \frac{(x^3 + 1)e^{-4x}}{y^{-2}}. & 19.3. & y' + y = x\sqrt{y}. \\
 19.4. & y' - y = \frac{x}{y} e^{2x}. & 19.5. & y' - \frac{2xy}{1+x^2} = \sqrt{y} \cdot \operatorname{arctg} x. & 19.6. & y' + x = \frac{3}{\sqrt[3]{y^2}}. \\
 19.7. & y' + \frac{2y}{x} = \frac{2\sqrt{y}}{\cos^2 x}. & 19.8. & 2x^3 y y' + 3x^2 y^2 + 1 = 0. & 19.9. & y' + y = \frac{x}{y^2}. \\
 19.10. & xy' + y = y^2 \ln x. & 19.11. & x(x-1) \cdot y' + y^3 = x \cdot y. & 19.12. & \frac{y'}{\sqrt{y}} = x + \frac{x\sqrt{y}}{x^2 - 1}. \\
 19.13. & y' + y^2 \cos x = y. & 19.14. & y' + y^2 \cdot \cos x = y \cdot \operatorname{tg} x. & 19.15. & \frac{y'}{y^3} + \frac{2x}{y^2} = 2x^3. \\
 19.16. & y' = \frac{y^2}{2} - \frac{y}{2x}. & 19.17. & y' - \frac{3y}{2x} = -\frac{(5x^2 + 3)y^3}{2x}. & 19.18. & \frac{y'}{x} + \frac{y}{x} = \frac{y^2}{2}. \\
 19.19. & y' = \frac{y^2 \ln x}{3x} - \frac{y}{3x}. & 19.20. & \frac{y'}{1-x^3} + \frac{4x^3 y}{1-x^3} = 4e^{4x} y^2. & 19.21. & y' - y = 2y^2 x. \\
 19.22. & y' + \frac{y}{x} = \frac{2y^2 \ln x}{x}. & 19.23. & \frac{y'}{x-1} + \frac{xy}{x-1} = e^x y^2. & 19.24. & \frac{y'}{x} - \frac{y}{x} = y^2. \\
 19.25. & y' + 2xy = 2x^3 y^3. & 19.26. & 2\left(\frac{y'}{x-1} + \frac{xy}{x-1}\right) = e^x y^2. & 19.27. & y' + y = y^2 x \\
 19.28. & y' + \frac{y}{3x} = \frac{y^2}{3}. & 19.29. & 3xy' + 5y = (4x-5)y^4. & 19.30. & y' + y = \frac{x}{y^2}
 \end{array}$$

20. Найти общий интеграл дифференциального уравнения.

$$\begin{array}{ll}
 20.1. & \left(1 + \frac{e^{\frac{x}{y}}}{y}\right) dx + \frac{y^2 - x}{y^2} e^{\frac{x}{y}} dy = 0. \\
 20.2. & (x^3 + xy^2) dx + (x^2 y + y^3) dy = 0. \\
 20.3. & y' = \frac{3x^2 - 2x - y}{x - 2y - 3y^2}. \\
 20.4. & \left(2x - 1 - \frac{y}{x^2}\right) dx - \left(2y - \frac{1}{x}\right) dy = 0. \\
 20.5. & y' = -\frac{3x^2 + 6xy^2}{6x^2 y + 4y^3}. \\
 20.6. & \frac{\sin 2x + xy}{y} dx + \frac{y^3 - \sin^2 x}{y^2} dy = 0. \\
 20.7. & \left(\frac{1}{x^2} + \frac{3y^2}{x^4}\right) dx - \frac{2y}{x^3} dy = 0. \\
 20.8. & y' = -\frac{3y^2 + 2xy + 2x}{6xy + x^3 + 3}. \\
 20.9. & y' = \frac{2x(1 + \sqrt{x^2 - y})}{\sqrt{x^2 - y}}. \\
 20.10. & (2x - y + 1) dx + (2y - x - 1) dy = 0.
 \end{array}$$

20.11. $e^{-y}dx + (1 - xe^{-y})dy = 0.$	20.12. $y' = \frac{2x \cos^2 y}{x^2 \sin 2y - 2y}.$
20.13. $e^y dx + (\cos y + xe^y)dy = 0.$	20.14. $y' = \frac{x^2 - 3xy^2}{3x^2y - 6y^2 - 1}.$
20.15. $(\ln y - 2x)dx + \frac{x - 2y^2}{y}dy = 0.$	20.16. $\frac{2xdx}{y^3} + \frac{y^2 - 3x^2}{y^4}dy = 0.$
20.17. $3 \cdot x^2 \cdot e^y + (x^3 \cdot e^y - 1) \cdot y' = 0.$	20.18. $(2x^3 - xy^2)dx + (2y^3 - x^2y)dy = 0.$
20.19. $\frac{xdy}{x^2 + y^2} = \left(\frac{y}{x^2 + y^2} - 1 \right) dx.$	20.20. $e^y dx + (x \cdot e^y - 2 \cdot y)dy = 0.$
20.21. $\frac{1 + xy}{x^2y}dx + \frac{1 - xy}{xy^2}dy = 0.$	20.22. $\frac{dx}{y} - \frac{x + y^2}{y^2}dy = 0.$
20.23. $\frac{dy}{y} - \frac{x + y^2}{y^2}dy = 0.$	20.24. $\left(xe^x + \frac{1}{x^2} \right) dx - \frac{dy}{x} = 0.$
20.25. $\operatorname{tg} x dx + \left(\frac{y - \ln(\cos x)}{y} \right) dy = 0.$	20.26. $y' = -\frac{e^x \sin y + x}{e^x \cos y + y}.$
20.27. $(3x^2y^2 + 7)dx + 2x^3ydy = 0.$	20.28. $(3x^2 + 4y^2)dx + (8xy + e^y)dy = 0.$
20.29. $(xy^2 - x^3)dx + (x^2y - y)dy = 0.$	20.30. $xe^{y^2}dx + \left(x^2ye^{y^2} + \operatorname{tg}^2 y \right) dy = 0$

21. Найти общее решение или общий интеграл дифференциального уравнения.

21.1. $xy'' + y' = \ln \frac{y'}{x}.$	21.12. $(1 + x^2)y'' + 2xy' = 12x^3.$	21.3. $xy''' - y'' = 0.$
21.4. $xy'' - y' = x^2e^x.$	21.5. $(x + 1)y''' + y'' = (x + 1).$	21.6. $xy'' - y' = x^2.$
21.7. $x^3y'' + x^2y' = 1.$	21.8. $y'' - 2y' \operatorname{ctg} x = \sin^3 x.$	21.9. $y'' + y' = \sin x.$
21.10. $xy'' + y' = \ln x.$	21.11. $(x^2 + 1)y'' + 2xy' = x^3.$	21.12. $y'' = \frac{y'}{x} + \frac{x^2}{y'}.$
21.13. $xy''' - 2y'' = -\frac{2}{x^2}.$	21.14. $y'' - \frac{y'}{x-1} = x^2 - x.$	21.15. $y'' = y' + x.$
21.16. $x^4y'' + x^3y' = 4.$	21.17. $y'' + \frac{2x}{x^2 + 1}y' = 2x.$	21.18. $xy'' = y' \ln \frac{y'}{x}.$
21.19. $xy''' + y'' = \sqrt{x}.$	21.20. $(1 + x^2)y'' - 2xy' = 0.$	21.21. $y'' \operatorname{tg} x - y' = 1.$
21.22. $y''' + \frac{y''}{x} = \frac{1}{\sqrt{x^5}}.$	21.23. $y''(e^x + 1) + y' = 0.$	21.24. $x^2y'' + xy' = 1.$
21.25. $xy''' - y'' + \frac{1}{x} = 0.$	21.26. $xy'' + xy'^2 - y' = 0.$	21.27. $xy''' + y'' = \frac{1}{\sqrt{x}}.$
21.28. $2xy'y'' - y'^2 = 1.$	21.29. $y'' + y' \operatorname{tg} x = \sin 2x.$	21.30. $y'' \ln x - \frac{y'}{x} = 0.$

22. Решить задачу Коши при заданных начальных условиях.

22.1. $y''' = y''^2$,
 $y''(0) = -1, y'(0) = 0, y(0) = 1.$

22.3. $y''^2 = 4(y' - 1)$,
 $y(0) = 0, y'(0) = 2.$

22.5. $y''y^3 = 1$,
 $y(0) = 1, y'(0) = 1.$

22.7. $yy'' - y'^2 = 0$,
 $y(0) = 1, y'(0) = 2.$

22.9. $2y'^2 = (y - 1)y''$,
 $y(0) = y'(0) = 2.$

22.11. $y'' = \frac{1}{\sqrt{y}}$,
 $y(0) = y'(0) = 0.$

22.13. $y'' = y' / \sqrt{y}$,
 $y(0) = 1, y'(0) = 2.$

22.15. $y(1 - \ln y)y'' + (1 + \ln y)y'^2 = 0$,
 $y(0) = y'(0) = 1.$

22.17. $y'' - y'^2 - y' = 0$,
 $y(0) = 0, y'(0) = 1.$

22.19. $2y'^2 = yy''$,
 $y(0) = 1, y'(0) = 2.$

22.21. $2yy'' - y'^2 + 1 = 0$,
 $y(0) = 2, y'(0) = 1.$

22.23. $y'' + y'^2 - 1 = 0$,
 $y(0) = 0, y'(0) = 0.$

22.25. $y'' = (1 + y'^2)^{3/2}$,
 $y(0) = 1, y'(0) = 0.$

22.27. $e^{-y} \cdot y'' - y' = 0$,
 $y(0) = 0, y'(0) = 1.$

22.29. $y'' + 50 \cdot \sin y \cdot \cos^3 y = 0$,
 $y(0) = 0, y'(0) = 5.$

22.2. $2yy'' = y'^2$,
 $y(-1) = 4, y'(-1) = 1.$

22.4. $yy'' - y'^2 = y^3$,
 $y(0) = -0,5, y'(0) = 0.$

22.6. $yy'' - y'^2 = y^2 \ln y$,
 $y(0) = y'(0) = 1.$

22.8. $y'' + yy'^3 = 0$,
 $y(0) = 1, y'(0) = 2.$

22.10. $4y''^2 = y'^2 + 1$,
 $y(0) = 1, y'(0) = 0.$

22.12. $yy'' - 2yy' \ln y = y'^2$,
 $y(0) = y'(0) = 1.$

22.14. $y''(1 + y) = y'^2 + y'$,
 $y(0) = y'(0) = 2.$

22.16. $y'' = -\frac{2}{1 - y} y'^2$,
 $y(0) = 0, y'(0) = 1.$

22.18. $y'' + yy'' - 5y'^2 = 0$,
 $y(0) = 0, y'(0) = 1.$

22.20. $y'' = y^{-3}$,
 $y(0) = 1, y'(0) = 0.$

22.22. $y''^2 - y' = 0$,
 $y(0) = 2/3, y'(0) = 1.$

22.24. $2yy'' - y'^2 = 0$,
 $y(0) = 1, y'(0) = 1.$
 $y'' - 2\sin^3 y \cdot \cos y = 0,$

22.26. $y(1) = \frac{\pi}{2}, y'(1) = 1.$

22.28. $y \cdot y'' - 2 \cdot y \cdot y' \cdot \ln y = y^2$,
 $y(0) = 1, y'(0) = 1.$

22.30. $y'' = -yy'^3$,
 $y(0) = 1, y'(0) = 2.$

23. Решить задачу Коши при заданных начальных условиях.

23.1. $y^{IV} + 10y'' + 9y = 0$, $y(0) = 1$, $y'(0) = 3$, $y''(0) = -9$, $y'''(0) = -27$.

23.2. $y^{IV} + 5y'' + 4y = 0$, $y(0) = 1$, $y'(0) = 4$, $y''(0) = -1$, $y'''(0) = -16$.

23.3. $y''' - y'' - y' + y = 0$, $y(0) = -1$, $y'(0) = 0$, $y''(0) = 1$.

23.4. $y''' + 2y'' + y' = 0$, $y(0) = 1$, $y'(0) = 2$, $y''(0) = -3$.

23.5. $y^V - 6y^{IV} + 9y''' = 0$, $y(0) = y'(0) = y''(0) = y'''(0) = 0$, $y^{IV}(0) = 27$.

23.6. $y''' + 2y'' + 9y' + 18y = 0$, $y(0) = 1$, $y'(0) = -3$, $y''(0) = -9$.

23.7. $y''' + y'' - 4y' - 4y = 0$, $y(0) = 0$, $y'(0) = 0$, $y''(0) = 12$.

23.8. $y^{IV} - 16y = 0$, $y(0) = 0$, $y'(0) = 0$, $y''(0) = 0$, $y'''(0) = -8$.

23.9. $y^{IV} - y = 0$, $y(0) = y'(0) = y''(0) = 0$, $y'''(0) = -4$.

23.10. $y^{IV} - 2y''' + y'' = 0$, $y(0) = y'(0) = 0$, $y''(0) = 1$, $y'''(0) = 2$.

23.11. $y''' - y'' + 4y' - 4y = 0$, $y(0) = -1$, $y'(0) = 0$, $y''(0) = -6$.

23.12. $y''' - 3y'' + 3y' - y = 0$, $y(0) = 0$, $y'(0) = 0$, $y''(0) = 4$.

23.13. $y''' - y'' + y' - y = 0$, $y(0) = 0$, $y'(0) = 1$, $y''(0) = 0$.

23.14. $y^{IV} - 10y'' + 9y = 0$, $y(0) = 0$, $y'(0) = 0$, $y''(0) = -8$, $y'''(0) = 24$.

23.15. $y^{IV} - 5y'' + 4y = 0$, $y(0) = -2$, $y'(0) = 1$, $y''(0) = 2$, $y'''(0) = 0$.

23.16. $y''' - 13y'' + 12y' = 0$, $y(0) = 0$, $y'(0) = 1$, $y''(0) = 133$.

23.17. $y''' + 9y' = 0$, $y(0) = 0$, $y'(0) = 9$, $y''(0) = -18$.

23.18. $y''' - 2y'' + 9y' - 18y = 0$, $y(0) = -2.5$, $y'(0) = 0$, $y''(0) = 0$.

23.19. $y''' + 3y'' + 3y' + y = 0$, $y(0) = -1$, $y'(0) = 0$, $y''(0) = 1$.

23.20. $y''' + 3y'' + 2y' = 0$, $y(0) = 0$, $y'(0) = 0$, $y''(0) = 2$.

23.21. $y''' - 5y'' + 8y' - 4y = 0$, $y(0) = 1$, $y'(0) = -1$, $y''(0) = 0$.

23.22. $y''' + y'' = 0$, $y(0) = 0$, $y'(0) = 1$, $y''(0) = -1$.

23.23. $y''' + y'' - 5y' + 3y = 0$, $y(0) = 0$, $y'(0) = 1$, $y''(0) = -14$.

23.24. $y^{IV} + 2y''' - 2y' - y = 0$, $y(0) = 0$, $y'(0) = 0$, $y''(0) = 0$, $y'''(0) = 8$.

23.25. $y''' - y' = 0$, $y(0) = 0$, $y'(0) = 2$, $y''(0) = 4$.

23.26. $y''' + y' = 0$, $y(0) = 0$, $y'(0) = 1$, $y''(0) = 1$.

23.27. $y''' - 4y' = 0$, $y(0) = 0$, $y'(0) = 2$, $y''(0) = 4$.

23.28. $y''' - y'' = 0$, $y(0) = 0$, $y'(0) = 0$, $y''(0) = -1$.

23.29. $y^V - 9y''' = 0$, $y(0) = 1$, $y'(0) = -1$, $y''(0) = 0$, $y'''(0) = 0$, $y^{IV}(0) = 0$.

23.30. $y''' - 7y'' + 6y' = 0$, $y(0) = y'(0) = 0$, $y''(0) = 30$.

24. Найдите общее решение уравнения. Решить задачу Коши при заданных начальных условиях.

24.1. $y'' - 2y' + y = 6xe^x$.

24.2. $y'' + 2y' + 5y = 17\sin 2x$.

24.3. $y'' - 2y' + 10y = 37\cos 3x$.

24.4. $y'' - y = -4\cos x + 2\sin x$.

24.4. $y'' - y' - 6y = 9\cos x - \sin x$.

24.6. $y'' - y' + y = -13\sin 2x$.

24.7. $y'' + 8y' + 25y = 18e^{5x}$.

24.8. $2y'' + 7y' + 3y = 222\sin 3x$.

- 24.9. $6y'' - y' - y = 3e^{2x}$.
 24.11. $y'' + 2y' + 37y = 37x^2 - 33x + 74$.
 24.13. $y'' + 6y' + 5y = 25x^2 - 2$.
 24.15. $y'' - 4y' + 29y = 104\sin 5x$.
 24.17. $y'' - 5y' + 4y = 4x^2e^{2x}$.
 24.19. $y'' + 9y = 9x^4 + 12x^2 - 27$.
 24.21. $y'' - 6y' + 13y = -3\cos 2x$.
 24.23. $y'' + y = 2\cos 5x + 3\sin 5x$.
 24.25. $y''' - y'' = 12x^2 + 6x$.
 24.27. $y'' + y = 2\cos 3x - 3\sin 3x$.
 24.29. $y'' + 2y' - 3y = x^2e^x$.
 24.10. $y'' + y = 4xe^x$.
 24.12. $y'' - 4y' + 5y = (3\sin x + \cos x)e^{-2x}$.
 24.14. $y'' + y' + y = 6e^{-x}$.
 24.16. $y'' + 16y = 8\cos 4x$.
 24.18. $y'' - 12y' + 40y = 2e^{6x}$.
 24.20. $y'' + 4y' = e^x(12\cos 2x + \sin 2x)$.
 24.22. $y'' - 3y' + 2y = 5\sin 2x$.
 24.24. $y'' - 4y' + 4y = e^{2x}\sin 5x$.
 24.26. $y'' + 2y' + 5y = -\cos x$.
 24.28. $y'' - 8y' = 16 + 48x^2 - 128x^3$.
 24.30. $y'' - 9y' + 18y = 26\cos x - 8\sin x$.

25. Решить задачу Коши при заданных начальных условиях.

- 25.1. $y'' + y' = 2x\cos x$,
 $y(0) = 1, y'(0) = 0$.
 25.2. $y'' - y' = 2(1 - x)$,
 $y(0) = y'(0) = 1$.
 25.3. $y''' - y' = 6 - 3x^2$,
 $y(0) = y'(0) = y''(0) = 1$.
 25.4. $y'' - 4y' + 5y = xe^{2x}$,
 $y(0) = -1, y'(0) = 0$.
 25.5. $y'' - 2y' + 5y = e^{2x}$,
 $y(0) = 0, y'(0) = 4$.
 25.6. $y'' - 4y' + 4y = e^{2x}$,
 $y(0) = 0, y'(0) = 8$.
 25.7. $y'' - 6y' + 9y = 9x^2 - 12x + 2$,
 $y(0) = 1, y'(0) = 3$.
 25.8. $y'' - 10y' + 25y = (2x - 1)e^{5x}$,
 $y(0) = 1, y'(0) = 6$.
 25.9. $y'' + 4y' + 5y = 5x^2 - 32x + 5$,
 $y(0) = y'(0) = 0$.
 25.10. $y'' + 9y = 15\sin 2x$,
 $y(0) = -7, y'(0) = 0$.
 25.11. $y'' + y' = 2x^2e^x$,
 $y(0) = 5, y'(0) = 0,5$.
 25.12. $y'' + 3y' + 2y = 1 + x + x^2$,
 $y(0) = 0, y'(0) = 1$.
 25.13. $y'' + 9y = \cos 3x$,
 $y(0) = 1, y'(0) = 0$.
 25.14. $y'' + 2y' + y = \cos x$,
 $y(0) = y'(0) = 0$.
 25.15. $y'' + y' = 2x + x^2$,
 $y(0) = 4, y'(0) = -2$.
 25.16. $y'' - 9y = e^{-2x}$,
 $y(0) = y'(0) = 0$.
 25.17. $y''' + y'' = \sin x$,
 $y(0) = y'(0) = 1, y''(0) = 0$.
 25.18. $y''' - 2y'' + y' = 4$,
 $y(0) = 1, y'(0) = 2, y''(0) = -2$.
 25.19. $y'' + y' - 2y = \cos x - 3\sin x$,
 $y(0) = 1, y'(0) = 2$.
 25.20. $4y'' - 4y' + y = e^{x/2}$,
 $y(0) = -2, y'(0) = 0$.

$$25.21. \quad y'' - 2y' = e^x(x^2 + x - 3), \\ y(0) = 2, y'(0) = 2.$$

$$25.23. \quad y'' - 10y' + 25y = (x^2 + 8x)e^{5x}, \\ y(0) = 1, y'(0) = 0.$$

$$25.25. \quad y'' + 9y = 36e^{3x}, \\ y(0) = 2, y'(0) = 6.$$

$$25.27. \quad y'' - 5y' + 6y = (12x - 7)e^{-x}, \\ y(0) = y'(0) = 0.$$

$$25.29. \quad y'' - 3y' + 2y = 5\sin 2x, \\ y(0) = 3/4, y'(0) = -1/2.$$

$$25.22. \quad y'' - 8y' + 16y = (2x - 3)e^{4x}, \\ y(0) = y'(0) = 0.$$

$$25.24. \quad y'' + 4y = 4e^{7x}, \\ y(0) = y'(0) = 0.$$

$$25.26. \quad y'' - 8y' + 16y = (x + 6)e^{4x}, \\ y(0) = 0, y'(0) = 1.$$

$$25.28. \quad y'' - 2y' + y = 16e^x, \\ y(0) = 1, y'(0) = 2.$$

$$25.30. \quad y'' - 4y' = 6x^2 + 1, \\ y(0) = 2, y'(0) = 3.$$

26. Решить задачу Коши при заданных начальных условиях.

$$26.1. \quad y'' + y = \frac{1}{\cos x}, \\ y(0) = 1, y'(0) = 0.$$

$$26.3. \quad y'' + y = \frac{1}{\sin x}, \\ y(\pi/2) = 1, y'(\pi/2) = \pi/2. \\ y'' + y = 2\operatorname{ctg} x,$$

$$26.5. \quad y\left(\frac{\pi}{2}\right) = 1, y'\left(\frac{\pi}{2}\right) = 2.$$

$$26.7. \quad y'' + 3y' + 2y = \frac{e^{-x}}{(2 + e^x)}, \\ y(0) = y'(0) = 0.$$

$$26.9. \quad y'' + 6y' + 8y = \frac{4e^{-2x}}{2 + e^{2x}}, \\ y(0) = 0, y'(0) = 0.$$

$$26.11. \quad y'' + 4y = \frac{4}{\sin 2x}, \\ y(\pi/4) = 2, y'(\pi/4) = \pi.$$

$$26.13. \quad y'' + 0,25 \cdot y = 0,25 \cdot \operatorname{ctg}(0,5 \cdot x), \\ y(\pi) = 2, y'(\pi) = 0,5.$$

$$26.2. \quad y'' - 3y' + 2y = \frac{e^x}{(1 + e^{-x})}, \\ y(0) = y'(0) = 0.$$

$$26.4. \quad y'' - 3y' + 2y = \frac{1}{(1 + e^{-x})}, \\ y(0) = 1 + 2\ln 2, y'(0) = 3\ln 2.$$

$$26.6. \quad y'' + 4y = \frac{4}{\sin 2x}, \\ y(0) = 2, y'(0) = 0.$$

$$26.8. \quad y'' + 9y = \frac{9}{\sin 3x}, \\ y(0) = 1, y'(0) = 0.$$

$$26.10. \quad y'' + 16y = \frac{16}{\sin 4x}, \\ y(\pi/8) = 3, y'(\pi/8) = 2\pi.$$

$$26.12. \quad y'' - 3y' + 2y = \frac{1}{(2 + e^{-x})}, \\ y(0) = 1 + 3\ln 3, y'(0) = 5\ln 3.$$

$$26.14. \quad y'' - y' = \frac{e^{-x}}{2 + e^{-x}}, \\ y(0) = \ln 27, y'(0) = \ln 9 - 1.$$

$$26.15. \quad y'' - y' = \frac{1}{1 + e^x},$$

$$y(0) = 1, y'(0) = 2.$$

$$26.17. \quad y'' + 9y = \frac{9}{\sin 3x},$$

$$y(\pi/6) = 4, y'(\pi/6) = 3\pi/2.$$

$$26.19. \quad y'' - 6y' + 8y = \frac{4}{2 + e^{-2x}},$$

$$y(0) = 1 + 3\ln 3, y'(0) = 10\ln 3.$$

$$26.21. \quad y'' + 4y = 8\operatorname{ctg} 2x,$$

$$y(\pi/4) = 5, y'(\pi/4) = 4.$$

$$26.23. \quad y'' + \pi^2 y = \frac{\pi^2}{\sin \pi x},$$

$$y(1/2) = 1, y'(1/2) = \pi^2/2.$$

$$26.25. \quad y'' + \frac{1}{\pi^2} y = \frac{1}{\pi^2 \cos \frac{x}{\pi}},$$

$$y(0) = 2, y'(0) = 0.$$

$$26.27. \quad y'' - 6y' + 8y = \frac{4e^{2x}}{2 + e^{-2x}},$$

$$y(0) = y'(0) = 0.$$

$$26.29. \quad y'' - 9y' + 18y = \frac{9e^{3x}}{1 + e^{-3x}},$$

$$y(0) = y'(0) = 0.$$

$$26.16. \quad y'' + y = 4\operatorname{ctg} x,$$

$$y(\pi/2) = 4, y'(\pi/2) = 4.$$

$$26.18. \quad y'' + 16y = \frac{16}{\sin 4x},$$

$$y(0) = 3, y'(0) = 0.$$

$$26.20. \quad y'' + \pi^2 y = \frac{\pi^2}{\cos \pi x},$$

$$y(0) = 3, y'(0) = 0.$$

$$26.22. \quad y'' + 3y' = \frac{9e^{3x}}{1 + e^{3x}},$$

$$y(0) = \ln 4, y'(0) = 3 - \ln 8.$$

$$26.24. \quad y'' - 6y' + 8y = \frac{4}{1 + e^{-2x}},$$

$$y(0) = 1 + 2\ln 2, y'(0) = 6\ln 2.$$

$$26.26. \quad y'' + 4y = 4\operatorname{ctg} 2x,$$

$$y(\pi/4) = 3, y'(\pi/4) = 2.$$

$$26.28. \quad y'' + y = -\operatorname{ctg}^2 x,$$

$$y(\pi/2) = 2, y'(\pi/2) = 0.$$

$$26.30. \quad y'' + y = \operatorname{tg}^2 x,$$

$$y(0) = -2, y'(0) = 0.$$

27. Найти общее решение системы однородных линейных дифференциальных уравнений с постоянными коэффициентами двумя способами: а) методом исключения; б) используя характеристическое уравнение матрицы указанной системы.

$$27.1. \begin{cases} \dot{x} = 5x + 2y, \\ \dot{y} = 4x + 3y. \end{cases}$$

$$27.2. \begin{cases} \dot{x} = 4x - 12y, \\ \dot{y} = x - 4y. \end{cases}$$

$$27.3. \begin{cases} \dot{x} = 6x + 2y, \\ \dot{y} = 4x + 4y. \end{cases}$$

$$27.4. \begin{cases} \dot{x} = 4x + 12y, \\ \dot{y} = -x - 4y. \end{cases}$$

$$27.5. \begin{cases} \dot{x} = 14x + 2y, \\ \dot{y} = 4x + 12y. \end{cases}$$

$$27.6. \begin{cases} \dot{x} = 4x - 6y, \\ \dot{y} = 2x - 4y. \end{cases}$$

$$27.7. \begin{cases} \dot{x} = 12x + 2y, \\ \dot{y} = 4x + 10y. \end{cases}$$

$$27.8. \begin{cases} \dot{x} = 2x - 3y, \\ \dot{y} = -x - 2y. \end{cases}$$

$$27.9. \begin{cases} \dot{x} = 10x + 2y, \\ \dot{y} = 4x + 8y. \end{cases}$$

$$27.10. \begin{cases} \dot{x} = 4x - 6y, \\ \dot{y} = 2x - 3y. \end{cases}$$

$$27.11. \begin{cases} \dot{x} = -y, \\ \dot{y} = -3x - 2y. \end{cases}$$

$$27.12. \begin{cases} \dot{x} = 2x - 3y, \\ \dot{y} = x - 2y. \end{cases}$$

$$27.13. \begin{cases} \dot{x} = -x + 8y, \\ \dot{y} = x + y. \end{cases}$$

$$27.14. \begin{cases} \dot{x} = 4x + 6y, \\ \dot{y} = -2x - 4y. \end{cases}$$

$$27.15. \begin{cases} \dot{x} = x + y, \\ \dot{y} = -2x + 4y. \end{cases}$$

$$27.16. \begin{cases} \dot{x} = 3x - 8y, \\ \dot{y} = x - 3y. \end{cases}$$

$$27.17. \begin{cases} \dot{x} = -5x + 2y, \\ \dot{y} = x - 6y. \end{cases}$$

$$27.18. \begin{cases} \dot{x} = 2x - y, \\ \dot{y} = 2x - y. \end{cases}$$

$$27.19. \begin{cases} \dot{x} = -2x - 4y, \\ \dot{y} = -x + y. \end{cases}$$

$$27.20. \begin{cases} \dot{x} = 4x + 4y, \\ \dot{y} = -3x - 4y. \end{cases}$$

$$27.21. \begin{cases} \dot{x} = 6x - 2y, \\ \dot{y} = x + 3y. \end{cases}$$

$$27.22. \begin{cases} \dot{x} = 3x + 8y, \\ \dot{y} = -x - 3y. \end{cases}$$

$$27.23. \begin{cases} \dot{x} = 8x - 3y, \\ \dot{y} = 2x + 3y. \end{cases}$$

$$27.24. \begin{cases} \dot{x} = 4x - 4y, \\ \dot{y} = 3x - 4y. \end{cases}$$

$$27.25. \begin{cases} \dot{x} = 10x - 8y, \\ \dot{y} = -x + 4y. \end{cases}$$

$$27.26. \begin{cases} \dot{x} = 10x + 11y, \\ \dot{y} = 4x + 3y. \end{cases}$$

$$27.27. \begin{cases} \dot{x} = 5x + 2y, \\ \dot{y} = 4x + 3y. \end{cases}$$

$$27.28. \begin{cases} \dot{x} = 7x + 10y, \\ \dot{y} = -2x - 5y. \end{cases}$$

$$27.29. \begin{cases} \dot{x} = 8x - 6y, \\ \dot{y} = 2x - 5y. \end{cases}$$

$$27.30. \begin{cases} \dot{x} = 3x - 2y, \\ \dot{y} = 4x - 3y. \end{cases}$$

РАЗДЕЛ 4 ОПЕРАЦИОННОЕ ИСЧИСЛЕНИЕ

28. Найти функцию-оригинал по заданному изображению.

28.1	a)	$\frac{1}{(p^2-1)^2};$	б)	$\frac{p-1}{p^2(p^2+1)};$	в)	$\frac{e^{-p}}{(p-1) \cdot (p^2+2p+2)}.$
28.2	a)	$\frac{p}{(p^2+1)^2};$	б)	$\frac{p+2}{p^2(p^2-1)};$	в)	$\frac{e^{2p}}{(p+2) \cdot (p^2-2p+2)}.$
28.3	a)	$\frac{4}{(p^2-4)^2};$	б)	$\frac{p-3}{p^2(p^2+4)};$	в)	$\frac{e^{-3p}}{(p-3) \cdot (p^2+4p+5)}.$
28.4	a)	$\frac{4p}{(p^2+4)^2};$	б)	$\frac{p+4}{p^2(p^2-4)};$	в)	$\frac{e^{4p}}{(p+4) \cdot (p^2-4p+5)}.$
28.5	a)	$\frac{1}{(p^2-9)^2};$	б)	$\frac{p-5}{p^2(p^2+9)};$	в)	$\frac{e^{-5p}}{(p-5) \cdot (p^2+2p+10)}.$
28.6	a)	$\frac{p}{(p^2+9)^2};$	б)	$\frac{p+6}{p^2(p^2-9)};$	в)	$\frac{e^{6p}}{(p+6) \cdot (p^2-2p+10)}.$
28.7	a)	$\frac{9}{(p^2-25)^2};$	б)	$\frac{p-7}{p^2(p^2+25)};$	в)	$\frac{e^{-7p}}{(p-7) \cdot (p^2+4p+13)}.$
28.8	a)	$\frac{9p}{(p^2+25)^2};$	б)	$\frac{p+8}{p^2(p^2-25)};$	в)	$\frac{e^{8p}}{(p+8) \cdot (p^2-4p+13)}.$
28.9	a)	$\frac{16}{(p^2-36)^2};$	б)	$\frac{p-9}{p^2(p^2+36)};$	в)	$\frac{e^{-9p}}{(p-9) \cdot (p^2+6p+13)}.$
28.10	a)	$\frac{10p}{(p^2+36)^2};$	б)	$\frac{p+10}{p^2(p^2-36)};$	в)	$\frac{e^{10p}}{(p+10) \cdot (p^2-6p+13)}.$
28.11	a)	$\frac{4}{(p^2-49)^2};$	б)	$\frac{p-11}{p^2(p^2+49)};$	в)	$\frac{e^{-11p}}{(p-11) \cdot (p^2+8p+25)}.$
28.12	a)	$\frac{12p}{(p^2+49)^2};$	б)	$\frac{p+12}{p^2(p^2-49)};$	в)	$\frac{e^{12p}}{(p+12) \cdot (p^2-8p+25)}.$
28.13	a)	$\frac{13}{(p^2-64)^2};$	б)	$\frac{p-13}{p^2(p^2+64)};$	в)	$\frac{e^{-13p}}{(p-13) \cdot (p^2+8p+20)}.$
28.14	a)	$\frac{p}{(p^2+64)^2};$	б)	$\frac{p+14}{p^2(p^2-64)};$	в)	$\frac{e^{14p}}{(p+14) \cdot (p^2-8p+20)}.$
28.15	a)	$\frac{15}{(p^2-100)^2};$	б)	$\frac{p-15}{p^2(p^2+100)};$	в)	$\frac{e^{-15p}}{(p-15) \cdot (p^2+10p+146)}.$
28.16	a)	$\frac{16p}{(p^2+100)^2};$	б)	$\frac{p+16}{p^2(p^2-100)};$	в)	$\frac{e^{16p}}{(p+16) \cdot (p^2-10p+146)}.$

28.17	a)	$\frac{17}{(p^2 - 121)^2};$	б)	$\frac{p - 17}{p^2(p^2 + 121)};$	в)	$\frac{e^{-19p}}{(p - 10) \cdot (p^2 + 12p + 40)}.$
28.18	a)	$\frac{18p}{(p^2 + 121)^2};$	б)	$\frac{p + 18}{p^2(p^2 - 121)};$	в)	$\frac{e^{18p}}{(p + 18) \cdot (p^2 - 12p + 40)}.$
28.19	a)	$\frac{19}{(p^2 - 144)^2};$	б)	$\frac{p - 19}{p^2(p^2 + 144)};$	в)	$\frac{e^{-19p}}{(p - 19) \cdot (p^2 + 4p + 29)}.$
28.20	a)	$\frac{20p}{(p^2 + 144)^2};$	б)	$\frac{p + 20}{p^2(p^2 - 144)};$	в)	$\frac{e^{20p}}{(p + 20) \cdot (p^2 - 4p + 29)}.$
28.21	a)	$\frac{21}{(p^2 - 169)^2};$	б)	$\frac{p - 21}{p^2(p^2 + 169)};$	в)	$\frac{e^{-21p}}{(p - 21) \cdot (p^2 + 12p + 40)}.$
28.22	a)	$\frac{22p}{(p^2 + 169)^2};$	б)	$\frac{p + 22}{p^2(p^2 - 169)};$	в)	$\frac{e^{22p}}{(p + 22) \cdot (p^2 - 12p + 40)}.$
28.23	a)	$\frac{23}{(p^2 - 196)^2};$	б)	$\frac{p - 23}{p^2(p^2 + 196)};$	в)	$\frac{e^{-23p}}{(p - 23) \cdot (p^2 + 14p + 53)}.$
28.24	a)	$\frac{24p}{(p^2 + 196)^2};$	б)	$\frac{p + 24}{p^2(p^2 - 196)};$	в)	$\frac{e^{24p}}{(p + 24) \cdot (p^2 - 14p + 53)}.$
28.25	a)	$\frac{25}{(p^2 - 225)^2};$	б)	$\frac{p - 25}{p^2(p^2 + 225)};$	в)	$\frac{e^{-25p}}{(p - 25) \cdot (p^2 + 2p + 65)}.$
28.26	a)	$\frac{26p}{(p^2 + 225)^2};$	б)	$\frac{p + 26}{p^2(p^2 - 225)};$	в)	$\frac{e^{26p}}{(p + 26) \cdot (p^2 - 2p + 65)}.$
28.27	a)	$\frac{27}{(p^2 - 256)^2};$	б)	$\frac{p - 27}{p^2(p^2 + 256)};$	в)	$\frac{e^{-27p}}{(p - 27) \cdot (p^2 + 4p + 85)}.$
28.28	a)	$\frac{28p}{(p^2 + 256)^2};$	б)	$\frac{p + 28}{p^2(p^2 - 256)};$	в)	$\frac{e^{28p}}{(p + 28) \cdot (p^2 - 4p + 85)}.$
28.29	a)	$\frac{29}{(p^2 - 289)^2};$	б)	$\frac{p - 29}{p^2(p^2 + 289)};$	в)	$\frac{e^{-29p}}{(p - 29) \cdot (p^2 + 2p + 229)}.$
28.30	a)	$\frac{30p}{(p^2 + 289)^2};$	б)	$\frac{p + 30}{p^2(p^2 - 289)};$	в)	$\frac{e^{30p}}{(p + 30) \cdot (p^2 - 2p + 229)}.$

29. Методами операционного исчисления решить задачу Коши для заданного дифференциального уравнения.

29.1 $x'' + x' - 2x = 2\sin t - 3\cos t,$

$x(0) = 2, x'(0) = 1.$

29.3 $x'' + 4x' + 5x = \operatorname{ch} 2t,$

$x(0) = 2, x'(0) = 3.$

29.5 $x'' - 5x' + 6x = 2\sin 3t + \cos 3t,$

$x(0) = 3, x'(0) = 4.$

29.2 $x'' + x = \operatorname{sh} 2t,$

$x(0) = 1, x'(0) = 2.$

29.4 $x'' - x' - 2x = 3\sin 2t - 2\cos 2t,$

$x(0) = 3, x'(0) = 2.$

29.6 $x'' + 4x = \operatorname{ch} 2t,$

$x(0) = 4, x'(0) = 3.$

- 29.7 $x'' - 4x' + 5x = \operatorname{ch} 3t$,
 $x(0) = 4, x'(0) = 5$.
- 29.9 $x'' + 4x' - 5x = 2\sin 5t - 3\cos 5t$,
 $x(0) = 5, x'(0) = 6$.
- 29.11 $x'' + 2x' + 2x = \operatorname{ch} 4t$,
 $x(0) = 6, x'(0) = 7$.
- 29.13 $x'' + 6x' + 8x = 7\sin 7t + 7\cos 7t$,
 $x(0) = 7, x'(0) = 8$.
- 29.15 $x'' - 2x' + 2x = \operatorname{ch} 5t$,
 $x(0) = 8, x'(0) = 9$.
- 29.17 $x'' - 6x' + 5x = \sin 9t + 4\cos 9t$,
 $x(0) = 3, x'(0) = 1$.
- 29.19 $x'' + 6x' + 13x = \operatorname{ch} 6t$,
 $x(0) = 2, x'(0) = 0$.
- 29.21 $x'' - 8x' + 7x = 4\sin 5t + 3\cos 5t$,
 $x(0) = 4, x'(0) = 5$.
- 29.23 $x'' - 6x' + 13x = \operatorname{ch} 7t$,
 $x(0) = 2, x'(0) = 4$.
- 29.25 $x'' - 8x' + 15x = 2\sin t - 3\cos t$,
 $x(0) = 3, x'(0) = 5$.
- 29.27 $x'' + 2x' + 17x = \operatorname{ch} 8t$,
 $x(0) = 1, x'(0) = 2$.
- 29.29 $x'' + 6x' + 9x = 3\sin 3t + 3\cos 3t$,
 $x(0) = 3, x'(0) = 3$.
- 29.8 $x'' + 5x' + 6x = \sin 4t - \cos 4t$,
 $x(0) = 5, x'(0) = 4$.
- 29.10 $x'' + 9x = \operatorname{sh} 4t$,
 $x(0) = 6, x'(0) = 5$.
- 29.12 $x'' - 6x' + 8x = 6\sin 6t - 6\cos 6t$,
 $x(0) = 7, x'(0) = 6$.
- 29.14 $x'' + 16x = \operatorname{ch} 5t$,
 $x(0) = 8, x'(0) = 7$.
- 29.16 $x'' + 7x' - 8x = 2\sin 8t - \cos 8t$,
 $x(0) = 9, x'(0) = 8$.
- 29.18 $x'' + 25x = \operatorname{sh} 6t$,
 $x(0) = 1, x'(0) = 3$.
- 29.20 $x'' + 6x' + 5x = -5\sin 2t + 3\cos 2t$,
 $x(0) = 3, x'(0) = 4$.
- 29.22 $x'' + 36x = 3\operatorname{ch} 7t$,
 $x(0) = 2, x'(0) = 7$.
- 29.24 $x'' + 8x' + 7x = 6\sin 5t - 6\cos 5t$,
 $x(0) = 5, x'(0) = 5$.
- 29.26 $x'' + 49x = \operatorname{sh} 8t$,
 $x(0) = 7, x'(0) = -7$.
- 29.28 $x'' + 8x' + 15x = \sin t - \cos t$,
 $x(0) = -3, x'(0) = -5$.
- 29.30 $x'' + 64x = \operatorname{ch} 9t$,
 $x(0) = -8, x'(0) = 8$.

30. Методами операционного исчисления найти решение дифференциального уравнения, удовлетворяющее заданным начальным условиям $x(0) = 0, x'(0) = 0$.

- 30.1 $x'' - x = \frac{4}{e^{2t} + e^{-2t} + 2}$.
- 30.2 $x'' + 2x' + x = \frac{t}{te^t + e^t}$.
- 30.3 $x'' - x' = \frac{e^{-t}}{1 + e^{-t}}$.
- 30.4 $x'' - x' = \frac{1}{e^{-t} + 1}$.
- 30.5 $x'' + 4x' + 4x = \frac{1}{(e^t + 2te^t)^2}$.
- 30.6 $x'' + 2x' = \frac{4}{e^{2t} + e^{-2t} + 2}$.
- 30.7 $x'' - x = \frac{2e^t}{1 + e^{2t}}$.
- 30.8 $x'' + 2x' + x = \frac{1}{e^t + t^2 e^t}$.

$$30.9 \quad x'' - 2x' + x = \frac{4e^t}{e^{2t} + e^{-2t} + 2}.$$

$$30.11 \quad x'' + x' = \frac{1}{2 + \frac{1}{2} \operatorname{ch} t}.$$

$$30.13 \quad x'' - 4x' + 4x = \frac{8e^{2t}}{e^{4t} + e^{-4t} + 2}.$$

$$30.15 \quad x'' - 9x = \frac{\operatorname{sh} 3t}{\operatorname{ch}^2 3t}.$$

$$30.17 \quad x'' - 4x = \frac{1}{\operatorname{ch}^3 2t}.$$

$$30.19 \quad x'' - 10x' + 25x = \frac{e^{5t}}{1 + t^2}.$$

$$30.21 \quad x'' - 9x = \operatorname{th}^2 3t.$$

$$30.23 \quad x'' + x' = \frac{e^{-2t}}{(1 + e^{-t})^2}.$$

$$30.25 \quad x'' - 16x = \operatorname{th}^2 4t.$$

$$30.27 \quad x'' - x' = \frac{1}{(1 + e^{-t})^2}.$$

$$30.29 \quad x'' - 25x = \frac{1}{\operatorname{ch}^2 5t}.$$

$$30.10 \quad x'' + x' = \frac{e^{-t}}{e^{-t} + 1}.$$

$$30.12 \quad x'' - 2x' + x = \frac{1}{e^{-t}(1 + t)}.$$

$$30.14 \quad x'' - 4x = \frac{1}{1 + \operatorname{ch} 2t}.$$

$$30.16 \quad x'' + 6x' + 9x = \frac{e^{-3t}}{(t + 3)^2}.$$

$$30.18 \quad x'' - 9x = \operatorname{th} 3t.$$

$$30.20 \quad x'' - 4x = \frac{2e^{2t}}{e^{4t} + 1}.$$

$$30.22 \quad x'' - x' = \frac{e^t}{2e^{-t} + 1}.$$

$$30.24 \quad x'' + 4x' + 4x = \frac{e^{-2t}}{\operatorname{ch}^2 2t}.$$

$$30.26 \quad x'' + x' = \frac{e^{2t}}{2 + e^t}.$$

$$30.28 \quad x'' + x' = \frac{1}{1 + e^{-t}}.$$

$$30.30 \quad x'' - 2x' = \frac{2}{1 + e^{-2t}}.$$

31. Методами операционного исчисления решить систему дифференциальных уравнений при заданных начальных условиях.

$$31.1 \quad \begin{cases} x' = 2x + y + e^t, \\ y' = 5x - 2y + 2e^t; \end{cases} \\ x(0) = 1, y(0) = -4.$$

$$31.2 \quad \begin{cases} x' = 3x + 7y + e^{2t}, \\ y' = x - 3y + 2e^{2t}; \end{cases} \\ x(0) = -2, y(0) = 3.$$

$$31.3 \quad \begin{cases} x' = 4x - 3y + 2e^{3t}, \\ y' = -x + 2y - 5e^{3t}; \end{cases} \\ x(0) = 3, y(0) = 5.$$

$$31.4 \quad \begin{cases} x' = x - y + 8e^{4t}, \\ y' = x + 3y - e^{4t}; \end{cases} \\ x(0) = 2, y(0) = 1.$$

$$\begin{aligned}
 31.5 \quad & \begin{cases} x' = -4x + 3y + 8e^{6t}, \\ y' = 3x + 4y + 2e^{6t}; \end{cases} \\
 & x(0) = 3, y(0) = -1.
 \end{aligned}$$

$$\begin{aligned}
 31.6 \quad & \begin{cases} x' = x + 4y - 6e^{5t}, \\ y' = 2x - y + 4e^{5t}; \end{cases} \\
 & x(0) = -1, y(0) = -2.
 \end{aligned}$$

$$\begin{aligned}
 31.7 \quad & \begin{cases} x' = 2x - 4y + 2e^{7t}, \\ y' = -x + 5y - 5e^{7t}; \end{cases} \\
 & x(0) = 4, y(0) = -1.
 \end{aligned}$$

$$\begin{aligned}
 31.8 \quad & \begin{cases} x' = 3x + 2y + 8e^{8t}, \\ y' = x + 2y - e^{8t}; \end{cases} \\
 & x(0) = 3, y(0) = 8.
 \end{aligned}$$

$$\begin{aligned}
 31.9 \quad & \begin{cases} x' = -5x - 3y + e^{9t}, \\ y' = 3x + 5y + 2e^{9t}; \end{cases} \\
 & x(0) = 4, y(0) = -2.
 \end{aligned}$$

$$\begin{aligned}
 31.10 \quad & \begin{cases} x' = 6x - 11y + e^{10t}, \\ y' = x - 6y + 2e^{10t}; \end{cases} \\
 & x(0) = -4, y(0) = -5.
 \end{aligned}$$

$$\begin{aligned}
 31.11 \quad & \begin{cases} x' = 4x - 3y + 3e^{-t}, \\ y' = x + 4e^{-t}; \end{cases} \\
 & x(0) = 2, y(0) = -9.
 \end{aligned}$$

$$\begin{aligned}
 31.12 \quad & \begin{cases} x' = -2y + 8e^{-2t}, \\ y' = x + 3y - e^{-2t}; \end{cases} \\
 & x(0) = 6, y(0) = -1.
 \end{aligned}$$

$$\begin{aligned}
 31.13 \quad & \begin{cases} x' = -7x + 4y + 8e^{-3t}, \\ y' = 8x + 7y + 2e^{-3t}; \end{cases} \\
 & x(0) = 2, y(0) = -5.
 \end{aligned}$$

$$\begin{aligned}
 31.14 \quad & \begin{cases} x' = 8x + 5y - 6e^{-4t}, \\ y' = -3x - 8y + 4e^{-4t}; \end{cases} \\
 & x(0) = -3, y(0) = -4.
 \end{aligned}$$

$$\begin{aligned}
 31.15 \quad & \begin{cases} x' = 3x - 2y + 2e^{-5t}, \\ y' = -2x + 6y - 2e^{-5t}; \end{cases} \\
 & x(0) = 6, y(0) = -3.
 \end{aligned}$$

$$\begin{aligned}
 31.16 \quad & \begin{cases} x' = 2x - 2y + 6e^{-6t}, \\ y' = 2x + 7y - 3e^{-6t}; \end{cases} \\
 & x(0) = -2, y(0) = -3.
 \end{aligned}$$

$$\begin{aligned}
 31.17 \quad & \begin{cases} x' = -9x + 19y + 2e^{-7t}, \\ y' = x + 9y + e^{-7t}; \end{cases} \\
 & x(0) = 2, y(0) = -3.
 \end{aligned}$$

$$\begin{aligned}
 31.18 \quad & \begin{cases} x' = 10x - 16y + e^{-8t}, \\ y' = 4x - 10y + 2e^{-8t}; \end{cases} \\
 & x(0) = -5, y(0) = 5.
 \end{aligned}$$

$$\begin{aligned}
 31.19 \quad & \begin{cases} x' = 3x - 5y + 7e^{-9t}, \\ y' = -x + 7y - e^{-9t}; \end{cases} \\
 & x(0) = 1, y(0) = -5.
 \end{aligned}$$

$$\begin{aligned}
 31.20 \quad & \begin{cases} x' = 4x - 2y + 8e^{-10t}, \\ y' = -6x + 5y - 3e^{-10t}; \end{cases} \\
 & x(0) = 5, y(0) = 10.
 \end{aligned}$$

$$\begin{aligned}
 31.21 \quad & \begin{cases} x' = -11x + 7y + 8e^{12t}, \\ y' = 3x + 11y + 2e^{12t}; \end{cases} \\
 & x(0) = -3, y(0) = -1.
 \end{aligned}$$

$$\begin{aligned}
 31.22 \quad & \begin{cases} x' = 12x + 4y - 6e^{11t}, \\ y' = 11x - 12y + 4e^{11t}; \end{cases} \\
 & x(0) = 1, y(0) = 2.
 \end{aligned}$$

$$\begin{aligned}
 31.23 \quad & \begin{cases} x' = 6x - 3y + 2e^{-11t}, \\ y' = -x + 4y - 5e^{-11t}; \end{cases} \\
 & x(0) = 8, y(0) = -5.
 \end{aligned}$$

$$\begin{aligned}
 31.24 \quad & \begin{cases} x' = 5x - 3y + 6e^{-12t}, \\ y' = -5x + 7y - e^{-12t}; \end{cases} \\
 & x(0) = 1, y(0) = -7.
 \end{aligned}$$

$$\begin{aligned}
 31.25 \quad & \begin{cases} x' = -13x - 3y + e^{13t}, \\ y' = 23x + 13y + 2e^{13t}; \end{cases} \\
 & x(0) = -2, y(0) = -2.
 \end{aligned}$$

$$\begin{aligned}
 31.26 \quad & \begin{cases} x' = 14x + 29y + e^{14t}, \\ y' = x - 14y + 2e^{14t}; \end{cases} \\
 & x(0) = -3, y(0) = -4.
 \end{aligned}$$

$$\begin{aligned}
 31.27 \quad & \begin{cases} x' = 7x - 3y + 2e^{-13t}, \\ y' = 4x + 3e^{-13t}; \end{cases} \\
 & x(0) = 1, y(0) = -5.
 \end{aligned}$$

$$\begin{aligned}
 31.28 \quad & \begin{cases} x' = -2y + 8e^{-14t}, \\ y' = -3x + 5y - e^{-14t}; \end{cases} \\
 & x(0) = 2, y(0) = -2.
 \end{aligned}$$

$$\begin{aligned}
 31.29 \quad & \begin{cases} x' = -16x - 12y + e^{-15t}, \\ y' = -12x + 16y + e^{-15t}; \end{cases} \\
 & x(0) = -1, y(0) = 1.
 \end{aligned}$$

$$\begin{aligned}
 31.30 \quad & \begin{cases} x' = 17x - 11y + e^{15t}, \\ y' = 3x - 17y + e^{15t}; \end{cases} \\
 & x(0) = -1, y(0) = -2.
 \end{aligned}$$